Denoising Deep Sparse Learning Based Channel Estimation for MU-MIMO Systems

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***Abstract*—Channel estimation for multi-user multiple input multiple output (MIMO) systems has been recognized as a key issue in next generation wireless communication. The wireless channel is approximately sparse due to the transmission noise effect, which limits the performance of the existing sparse channel estimation method. To address this problem, the denoising deep learning based channel estimation method for MU-MIMO system is proposed in this paper. Utilizing the denoising algorithm to remove noise perturbations in channel estimation, the proposed method can obtain the accurate sparse feature of wireless channels in MU-MIMO system. Moreover, the estimation accuracy and spectrum efficiency can be further improved by fully utilizing the spatial correlation among the massive MIMO channel. Simulation results demonstrated that, the proposed method can improve the channel estimation accuracy and robustness of massive MIMO channel compared with the existing benchmarks.**

***Index Terms*—massive MIMO, channel estimation, sparse recovery, denoising deep network**

I. Introduction

Massive multiple input multiple output has attracted much attention due to its advantage of meeting the demand of higher spectral efficiency and data rate in the 5G wireless communications [1] [2]. However, it is difficult to estimate the channel of massive MIMO systems accurately and efficiently with the dramatic increase of the scale of antenna [3] [4]. The channel estimation method with the reliable estimation performance for massive MIMO system has gradually become a research hotspot in recent years.

The conventional channel estimation methods can be summarized into the two types [5]: the time domain based method and the frequency domain based method. However, the overhead of time and frequency resources will dramatically increases when the scale of the antenna array becomes large, which significantly deteriorates the spectral efficiency of massive MIMO system [4]. To tackle this issue, with the help of the compressed sensing (CS) theory, the CS based channel estimation method is proposed to estimate the massive MIMO channel from a few received measurement data, which reduces the overhead and thus improves the spectral efficiency [6]–[8]. However, the accuracy of the existing CS based channel estimation methods is limited at the situation where the wireless channel is approximately sparse due to the transmission noise effect, i.e. the channel impulse response is a approximately sparse vector with several dominant taps and many smaller non-zero elements close to zero [9].

Recently, the deep learning (DL) has been adopted to deal with the classical problems in wireless communication due to its excellent performance in extracting the potential features of large amounts of data. Several deep leaning based methods are proposed to conduct the massive MIMO channel estimation [10]–[14]. By unrolling the repeated iterations of approximate message passing (AMP) algorithm into the multilayer neural networks, the learned AMP (LAMP) network based channel estimation method [15] is proposed to estimate the sparse channel vector in an end-to-end manner, which improves the channel estimation accuracy based on the capability of obtaining the optimal sparse feature of the LAMP network. However, the wireless channel is approximately sparse in the case of high noise intensity, the LAMP network based channel estimation method can not accurately learn the sparse feature, which results in low accuracy in channel estimation. Therefore, it is necessary to research an efficient method with higher accuracy to estimate the approximate sparse channel.

In this paper, the denoising deep sparse learning based channel estimation method (D-DSLCE) for MU-MIMO system is proposed to improve the estimation accuracy of approximate sparse channel. Utilizing the denoising algorithm to remove the noise perturbations, the proposed D-DSLCE method can efficiently and accurately obtain the sparse feature of massive MIMO channel, i.e. the dominant taps of the channel CIRs of MU-MIMO system. Moreover, the spatial correlation among a large number of antennas is also fully exploited to improve the estimation accuracy and spectrum efficiency. Simulation results demonstrate that the proposed D-DSLCE method achieves higher accuracy of MIMO channel estimation compared with the existing benchmark methods including the conventional least squares (LS) method, the CS-based method and the deep learning based method at the situation with high noise intensity.

II. System Model

As illustrated in Figure 1, we consider a massive MIMO system where the base station (BS) is equipped with N_t transmit antennas and serves multiple users with single-antenna. During a certain OFDM symbol in transmission, the L length-CIR

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between the t-th transmit antenna of the BS and a certain user can be modeled as

$$\mathbf{h}^{(t)}=\left[h_1^{(t)},h_2^{(t)},\cdots,h_L^{(t)}\right]^T \tag{1}$$

Due to the limited scattering points around the base station, the majority of the energy of the channel CIR is concentrated on a few dominant taps [16] [17]. Therefore, the CIR vector of wireless channel is approximately sparse vector with several dominant taps and many smaller non-zero elements close to zero in the delay domain. The time-frequency training OFDM signal structure of the t-th transmit antenna is composed of an M-length cyclic prefix (CP) $\mathbf{c}=[c_1,c_2,...,c_M]^T$ and an N-length OFDM symbol $\mathbf{x}_i$ given by

$$\mathbf{x}=\left[x_1^{(t)},x_2^{(t)},...,x_N^{(t)}\right]^T=\mathbf{F}^H\tilde{\mathbf{x}}^{(t)} \tag{2}$$

where $\mathbf{F}$ is the $N\times N$ discrete Fourier transform (DFT) matrix and N is the number of OFDM sub-carriers. $\tilde{\mathbf{x}}^{(t)}$ denotes the OFDM symbol in frequency domain, and N_p pilots are randomly distributed over the OFDM sub-carriers. The locations of the pilots are denoted by a set given by

$$D^{(t)}=\{d_n^{(t)}\}_{n=1}^{N_\mathrm{p}} \tag{3}$$

where $d_n^{(t)}$ is an index integer from 0 to $N-1$ denoting a pilot location. The pilots of different transmit antennas are distributed in the sub-carriers in an orthogonal pattern as illustrated in Figure 1.

At a certain user, the received frequency-domain OFDM symbol $\tilde{\mathbf{y}}\in\mathbb{C}^N$ can be represented as

$$\tilde{\mathbf{y}}=\sum_{t=1}^{N_t}\mathrm{diag}(\tilde{\mathbf{x}}^{(t)})\mathbf{F}_L\mathbf{h}^{(t)}+\tilde{\mathbf{w}} \tag{4}$$

where $\mathrm{diag}(\tilde{\mathbf{x}}^{(t)})$ is the diagonal matrix with the diagonal given by the vector $\tilde{\mathbf{x}}^{(t)}$, and $\mathbf{F}_L$ is the $N\times L$ partial DFT matrix composed of the first L columns of the $N\times N$ DFT matrix $\mathbf{F}$. $\tilde{\mathbf{w}}$ denotes the frequency-domain additive white Gaussian noise vector. Due to the pilots patterns of different transmit antennas are orthogonal to each other, the received pilots located at $D^{(t)}$ from the t-th transmit antenna can be extracted in the frequency domain, and represented as

$$\mathbf{u}^{(t)}=\mathbf{A}\mathbf{h}^{(t)}+\mathbf{w}^{(t)},\quad 1\le t\le N_t \tag{5}$$

where $\mathbf{u}^{(t)}=[\tilde{y}_{d_1}/\tilde{x}_{d_1}^{(t)},\tilde{y}_{d_2}/\tilde{x}_{d_2}^{(t)},\cdots,\tilde{y}_{d_{N_\mathrm{p}}}/\tilde{x}_{d_{N_\mathrm{p}}}^{(t)}]^T\in\mathbb{C}^{N_\mathrm{p}}$ denotes the receive pilot signals normalized by the transmitted original pilot power to represent channel measurements at the receiver, and the measurement matrix $\mathbf{A}$ is the $N_\mathrm{p}\times L$ partial DFT matrix with its entry in row-n and column-k being exp $(-j2\pi d_n^{(t)}(k-1)/N)/\sqrt{N}$.

Since the physical distance between different antennas in the base station is far less than the actual communication distance between the base station and the certain user in the long-distance communication, the CIR vectors of different transmit-receive antenna pairs have similar propagation path and characteristics [18] [19]. Therefore, the channel matrix of massive MIMO system can be regarded as a set of independent sub-channel vectors, which is given by

$$\mathbf{H}=\left[\mathbf{h}^{(1)},\mathbf{h}^{(2)},...,\mathbf{h}^{(N_t)}\right] \tag{6}$$

Moreover, the CIR vectors of different transmit-receive antenna pairs have the identical dominant taps, which is called as the spatial correlation of the MIMO channels [20]. Utilizing this property, the channel estimation accuracy can be further improved.

III. Proposed Denoising Deep Sparse Learning Based Channel Estimation Method

In this section, we explore the sparse characteristics of channel, i.e. the channel CIR vector has a few dominant taps in delay domain, and propose a denoising deep sparse learning based channel estimation method. Specifically, the massive MIMO channel estimation problem is formulated as a sparse signal recovery problem according to the Section 2, which can be solved by the iterative sparse recovery algorithm. Due to the transmission noise effect, the wireless channel is approximately sparse, which limits the accuracy of the existing sparse channel estimation method. Using the denoising algorithm to remove noise perturbations in channel estimation [21], the proposed D-DSLCE method utilizes the denoising deep network to learn the sparse feature of the massive MIMO channel and obtain the precise positions of the dominant taps, which improves the channel recovery performance of massive MIMO system compared with the existing benchmarks.

As illustrated in Figure 2, the denoising deep network consists of N_L deep neural layers connected by cascade way, and each layer is used to mimic an iteration of the iterative sparse recovery algorithm. Since the denoising algorithm in the deep network needs to be easy to propagate the gradient, DnCNN [22] with sufficiently good denoising performance is selected as the denoising algorithm in the denoising deep network. Given the received normalized pilot signals $\mathbf{U}=\left[\mathbf{u}^{(1)},\mathbf{u}^{(2)},...,\mathbf{u}^{(N_t)}\right]$ and the measurement matrix $\mathbf{A}$, the channel estimation process for the l layer of the denoising deep network can be represented as

$$\hat{\mathbf{H}}_{l+1}{=}D_{\sigma_l}\left(\hat{\mathbf{H}}_l{+}\mathbf{A}^T\mathbf{z}_l\right) \tag{7}$$

$$\mathbf{z}_{l+1}=\mathbf{U}-\mathbf{A}\hat{\mathbf{H}}_{l+1}+\frac{1}{N_\mathrm{p}}\mathbf{z}_l\mathrm{div}D_{\sigma_l}\left(\hat{\mathbf{H}}_l+\mathbf{A}^T\mathbf{z}_l\right) \tag{8}$$

where $\hat{\mathbf{H}}_l$ and $\hat{\mathbf{H}}_{l+1}$ are the MIMO CIR at the input and output of the l-th layer of the denoising deep network, respectively. $\mathbf{z}_l$ and $\mathbf{z}_{l+1}$ denote the iterative residual at the input and output of the l-th layer of the denoising deep network, respectively. The σ_l represents the standard deviation of iteration residuals. $D_{\sigma_l}\left(\right)$ denotes the denoising algorithm of DnCNN, whose input $\mathbf{r}_l=\hat{\mathbf{H}}_l+\mathbf{A}^T\mathbf{z}_l$ can be approximated as $\mathbf{r}_l=\mathbf{H}+\hat{\mathbf{n}}_l$, the equivalent noise $\hat{\mathbf{n}}_l=\hat{\mathbf{H}}_l-\mathbf{H}+\mathbf{A}^T\mathbf{z}_l\sim N\left(0,{\sigma_l}^2\mathbf{I}\right)$. Different layers of the denoising deep network adopt the identical DnCNN structure, which is used to estimate the channel CIR $\hat{\mathbf{H}}_l$ with the input of the intermediate quantities $\mathbf{r}_l$ and the learnable parameters $\Theta_l=\{\mathbf{w}_k,\mathbf{b}_k\}_{k=0}^{l}$. The equivalent noise

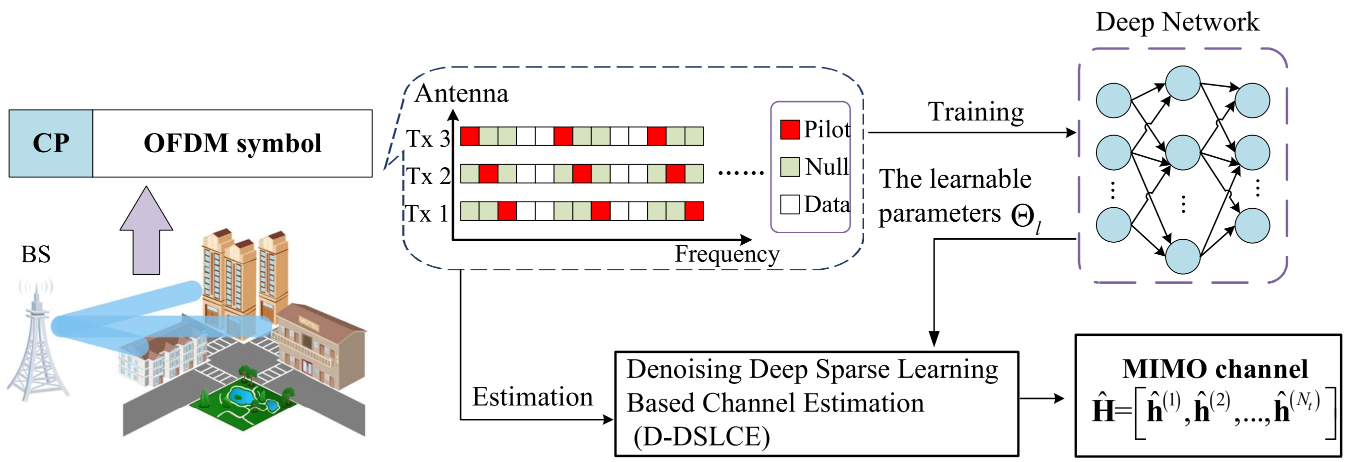

Fig. 1. The signal model of the MU-MIMO system and denoising deep sparse learning based channel estimation.

variance ${\sigma_l}^2$ is related to the iteration residuals $\mathbf{z}_l$, and will gradually decrease to a finite value with the increase of the number of layers in the denoising deep network. Besides, the Onsager correction term $\frac{1}{N_\mathrm{p}}\mathbf{z}_l \mathrm{div} D_{\sigma_l}\left(\hat{\mathbf{H}}_l + \mathbf{A}^T\mathbf{z}_l\right)$ in Equation (8) involves the divergence calculation of the denoising deep network. With the help of an independent and identically distributed random vector $b \sim N(0,\mathbf{I})$, Monte-Carlo method is used to compute the divergence

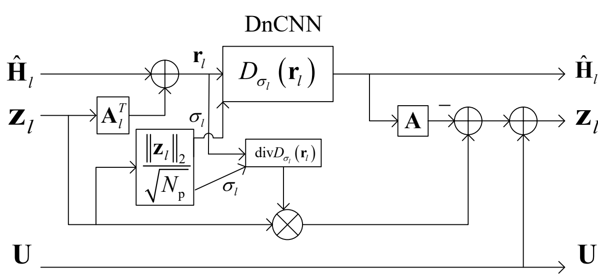

Fig. 2. The architecture of the layer-l of the denoising deep network

$$\mathrm{div} D_{\sigma_l}(x) = \lim_{\tau \to 0}\left\{ b^T\left(\frac{D_{\sigma_l}(x+\tau b) - D_{\sigma_l}(x)}{\tau}\right)\right\} \tag{9}$$

In order to optimize the depth of layers and learnable parameters of the denoising deep network, the loss function is determined as the normalized mean square error (NMSE) of the channel estimation, which is given by

$$\ell_l(\Theta_l) = \frac{1}{D}\sum_{d=1}^{D}\frac{\left\|\mathbf{H}_0^d - f_l\left(\mathbf{U}^d,\Theta_l\right)\right\|_2^2}{\left\|\mathbf{H}_0^d\right\|_2^2} \tag{10}$$

where $f_l(\mathbf{U}^d,\Theta_l)$ denotes the channel CIR $\hat{\mathbf{H}}_l$ estimated by the denoising deep network composed of l layers with input $\mathbf{U}^d$ and the learnable parameters Θ_l. The parameters Θ_l are learnt by minimizing the loss function over the training data set $\{\mathbf{U}^d\}_{d=1}^{D}$ in the training process.

With the trained denoising deep network, the MIMO channel can be estimated accurately in the subsequent prediction process. First, the received normalized pilot signals $\mathbf{U} = \left[\mathbf{u}^{(1)},\mathbf{u}^{(2)},...,\mathbf{u}^{(N_t)}\right]$ are input into the trained denoising deep network to estimate the channel CIR for each antenna respectively. Then, based on the sparse characteristic of massive MIMO channel, the dominant taps set $\Pi_S^{(t)}$ of each antenna can be formulated by selecting the corresponding subscript positions of the elements with the S largest amplitude value. S is the the upper bound of the channel CIR corresponding to the sub-channel in massive MIMO system [23]. Using the spatial correlation of the massive MIMO channel, the dominant taps set of each antenna can be further optimized by taking the intersection of the corresponding dominant taps set of each sub-channel. Therefore, the dominant taps set $\bar{\Pi}_S$ of massive MIMO channel can be obtained by

$$\bar{\Pi}_S = \bigcap_{t=1}^{N_t}\Pi_S^{(t)} \tag{11}$$

Finally, the MIMO channel matrix can be estimated by solving the LS problem in Equation (5):

$$\left(\hat{\mathbf{H}}\right)^T\Big|_{\bar{\Pi}_S} = \left(\mathbf{A}_{\bar{\Pi}_S}^{\dagger}\mathbf{U}\right)^T = \left(\left(\mathbf{A}_{\bar{\Pi}_S}^H\mathbf{A}_{\bar{\Pi}_S}\right)^{-1}\mathbf{A}_{\bar{\Pi}_S}^H\mathbf{U}\right)^T \tag{12}$$

IV. Simulation Results

In this section, we present the massive MIMO channel estimation performance of the proposed D-DSLCE method. As a comparison, the conventional LS based channel estimation method [4], the CS based algorithm of simultaneous orthogonal matching pursuit (SOMP) [24] and the state-of-the-art deep learning based method (LAMP) [15] are evaluated. Simulation parameters are summarized as follows. The antenna scale of the MIMO system is $N_t = N_r = 32$. The bandwidth is 8 MHz located at the central frequency of 780 MHz. The length of the OFDM data block is $N= 4096$, with the length of CP being $M= 256$. The maximum channel length is $L= 256$. The maximum sparsity level of the CIR is conservatively assumed to be $S= 9$, which is a conservative value to cover based on the statistical distribution of the sparse channel. The number of pilots adopted for channel measurements at each transmit antenna is set as $N_{\mathrm{P}}= 25$.

The training set $\left\{\mathbf{U}^{d}\in^{N_{\mathrm{P}}\times N_t}, \mathbf{H}_0^{d}\in^{L\times N_t}\right\}_{d=1}^{D}$, with $D = 2000$, is generated based on the parametric sparse channel model [25]. Each training sample in the training set containing a measurement $\mathbf{U}$ containing all the antennas and the corresponding ground-truth MIMO channel CIR vector $\mathbf{H}_0$. The dominant taps of the MIMO channel is randomly distributed in all the delay length and the amplitude follows a Rayleigh fading distribution. The test data set and the validation data set used to evaluate performance are generated in the same way as the training data set, and are independent from the training data set. The depth of network layers of D-DSLCE is determined according to the evaluation performance of the validation set. Each layer of the denoising deep network contains the DnCNN with 8 layers. The stochastic gradient descent method and Adam optimizer were used to train the network, and the learning rate was set at 0.001. The learning rate was set at 0.0001 to continue the training process when the loss error of validation set stopped declining.

Figure 3 presents the NMSE performance comparison of different methods under the multipath fading channel in 32×32 MIMO systems. It can be shown that, the performance of the proposed D-DSLCE method is obviously better than that of the LS based method. This is because the available pilot number of LS method is far less than the channel length, thus it can not solve the underdetermined problem in Equation (5) well. Besides, the proposed D-DSLCE method can effectively learn the channel sparse structure and accurately estimate the channel CIR vector under the same pilot number. At the level of NMSE=10^{-2}, the proposed D-DSLCE method has a SNR gain of about 3.5 dB and 1.5 dB compared with the SOMP based method and the LAMP based method respectively, which verifies the high accuracy of the proposed method.

Figure 4 shows the channel estimation performance of different methods with respect to the number of available pilots adopted for channel measurements when the SNR is 15dB. It can be shown that, the proposed D-DSLCE method has better estimation accuracy compared with the SOMP based method and the LAMP based method. Therefore, the proposed D-DSLCE method can use fewer pilots to achieve the same estimation accuracy as the benchmark methods, which reduces the overhead of pilot and improves the spectrum efficiency of massive MIMO system.

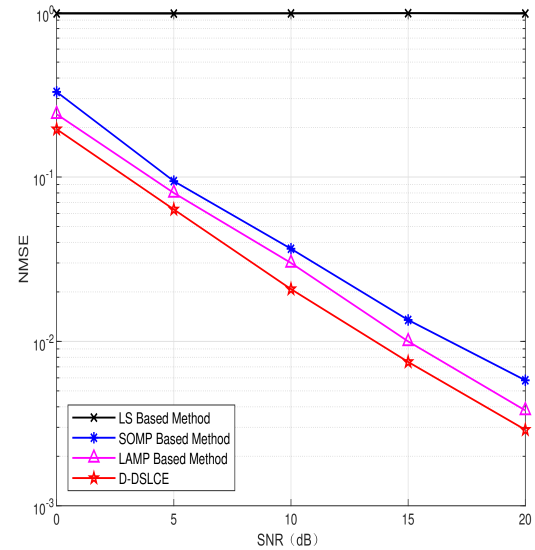

Fig. 3. NMSE performance comparison of the proposed D-DSLCE method and the state-of-the-art benchmark methods for 32×32 MIMO system.

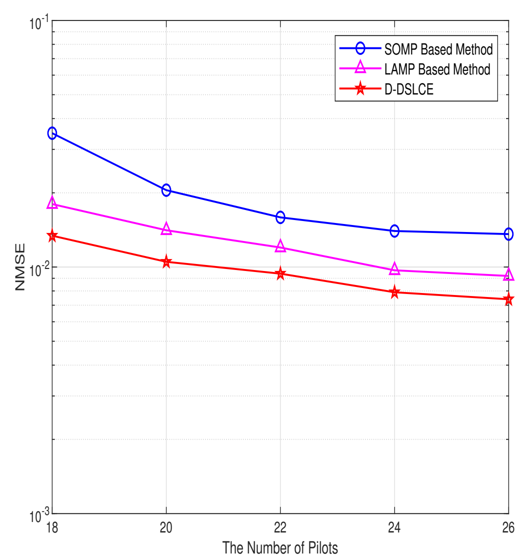

Fig. 4. Comparison of the NMSE performance with different number of pilots as channel measurement data at SNR = 15 dB.

V. Conclusion

In this paper, a novel denoising deep sparse learning based channel estimation method for massive MIMO system is proposed by exploiting the denoising deep network to estimate the sophisticated multipath fading MIMO channel with much higher accuracy. Simulation results have verified that, the proposed D-DSLCE method can significantly improve the estimation accuracy and spectrum efficiency of MIMO channel compared with the state-of-the-art benchmark schemes in realistic multi-users communications scenarios.

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References

[1] Lu Lu, Geoffrey Ye Li, A. Lee Swindlehurst, Alexei Ashikhmin, and Rui Zhang. An overview of massive mimo: Benefits and challenges. *IEEE Journal of Selected Topics in Signal Processing*, 8(5):742–758, 2014.

[2] Shangbin Wu, Cheng-Xiang Wang, Harald Haas, el-Hadi M. Aggoune, Mohammed M. Alwakeel, and Bo Ai. A non-stationary wideband channel model for massive mimo communication systems. *IEEE Transactions on Wireless Communications*, 14(3):1434–1446, 2015.

[3] Saif K. Mohammed, Ahmed Zaki, A. Chockalingam, and B. Sundar Rajan. High-rate spacetime coded large-mimo systems: Low-complexity detection and channel estimation. *IEEE Journal of Selected Topics in Signal Processing*, 3(6):958–974, 2009.

[4] Yasuhiro Takano and Hsuan-Jung Su. A low-complexity ls turbo channel estimation technique for mu-mimo systems. *IEEE Signal Processing Letters*, 25(5):710–714, 2018.

[5] Yinsheng Liu, Zhenhui Tan, Hongjie Hu, Leonard J. Cimini, and Geoffrey Ye Li. Channel estimation for ofdm. *IEEE Communications Surveys Tutorials*, 16(4):1891–1908, 2014.

[6] Yang Nan, Li Zhang, and Xin Sun. Efficient downlink channel estimation scheme based on block-structured compressive sensing for tdd massive mu-mimo systems. *IEEE Wireless Communications Letters*, 4(4):345–348, 2015.

[7] Kiran Venugopal, Ahmed Alkhateeb, Nuria Gonzlez Prelcic, and Robert W. Heath. Channel estimation for hybrid architecture-based wideband millimeter wave systems. *IEEE Journal on Selected Areas in Communications*, 35(9):1996–2009, 2017.

[8] Shuai Hou, Yafeng Wang, Tianyi Zeng, and Sheng Wu. Sparse channel estimation for spatial non-stationary massive mimo channels. *IEEE Communications Letters*, 24(3):681–684, 2020.

[9] Xu Ma, Fang Yang, Sicong Liu, Jian Song, and Zhu Han. Design and optimization on training sequence for mmwave communications: A new approach for sparse channel estimation in massive mimo. *IEEE Journal on Selected Areas in Communications*, 35(7):1486–1497, 2017.

[10] Hao Ye, Geoffrey Ye Li, and Biing-Hwang Juang. Power of deep learning for channel estimation and signal detection in ofdm systems. *IEEE Wireless Communications Letters*, 7(1):114–117, 2018.

[11] Yu Jin, Jiayi Zhang, Shi Jin, and Bo Ai. Channel estimation for cell-free mmwave massive mimo through deep learning. *IEEE Transactions on Vehicular Technology*, 68(10):10325–10329, 2019.

[12] Peihao Dong, Hua Zhang, Geoffrey Ye Li, Ivan Simes Gaspar, and Navid NaderiAlizadeh. Deep cnn-based channel estimation for mmwave massive mimo systems. *IEEE Journal of Selected Topics in Signal Processing*, 13(5):989–1000, 2019.

[13] Yinghui Zhang, Yifan Mu, Yang Liu, Tiankui Zhang, and Yi Qian. Deep learning-based beamspace channel estimation in mmwave massive mimo systems. *IEEE Wireless Communications Letters*, 9(12):2212–2215, 2020.

[14] Xiuhong Wei, Chen Hu, and Linglong Dai. Deep learning for beamspace channel estimation in millimeter-wave massive mimo systems. *IEEE Transactions on Communications*, 69(1):182–193, 2021.

[15] Mark Borgerding, Philip Schniter, and Sundeep Rangan. Amp-inspired deep networks for sparse linear inverse problems. *IEEE Transactions on Signal Processing*, 65(16):4293–4308, 2017.

[16] Zakia Jellali and Lela Najjar Atallah. Fast fading channel estimation by kalman filtering and cir support tracking. *IEEE Transactions on Broadcasting*, 63(4):635–643, 2017.

[17] Chao Wei, Huaping Liu, Zaichen Zhang, Jian Dang, and Liang Wu. Near-optimum sparse channel estimation based on least squares and approximate message passing. *IEEE Wireless Communications Letters*, 6(6):754–757, 2017.

[18] Yann Barbotin, Ali Hormati, Sundeep Rangan, and Martin Vetterli. Estimation of sparse mimo channels with common support. *IEEE Transactions on Communications*, 60(12):3705–3716, 2012.

[19] Mudassir Masood, Laila H. Afify, and Tareq Y. Al-Naffouri. Efficient coordinated recovery of sparse channels in massive mimo. *IEEE Transactions on Signal Processing*, 63(1):104–118, 2015.

[20] Zhiyuan Jiang, Andreas F. Molisch, Giuseppe Caire, and Zhisheng Niu. Achievable rates of fdd massive mimo systems with spatial channel correlation. *IEEE Transactions on Wireless Communications*, 14(5):2868–2882, 2015.

[21] C. A. Metzler, A. Mousavi, and R. G. Baraniuk. Learned D-AMP: Principled neural network based compressive image recovery. In *Proceedings of the 31st International Conference on Neural Information Processing Systems (NIPS'17)*, pages 1770–1781, 2017.

[22] Kai Zhang, Wangmeng Zuo, Yunjin Chen, Deyu Meng, and Lei Zhang. Beyond a gaussian denoiser: Residual learning of deep cnn for image denoising. *IEEE Transactions on Image Processing*, 26(7):3142–3155, 2017.

[23] Linglong Dai, Jintao Wang, Zhaocheng Wang, Paschalis Tsiaflakis, and Marc Moonen. Spectrum- and energy-efficient ofdm based on simultaneous multi-channel reconstruction. *IEEE Transactions on Signal Processing*, 61(23):6047–6059, 2013.

[24] Malong Ke, Zhen Gao, Yongpeng Wu, Xiqi Gao, and Robert Schober. Compressive sensing-based adaptive active user detection and channel estimation: Massive access meets massive mimo. *IEEE Transactions on Signal Processing*, 68:764–779, 2020.

[25] Thomas Lundgaard Hansen, Peter Bjrn Jrgensen, Mihai-Alin Badiu, and Bernard Henri Fleury. An iterative receiver for ofdm with sparsity-based parametric channel estimation. *IEEE Transactions on Signal Processing*, 66(20):5454–5469, 2018.