Clipping Noise Elimination for OFDM Systems by Compressed Sensing With Partially Aware Support

Fang Yang, *Senior Member, IEEE*, Junnan Gao, *Student Member, IEEE*, Sicong Liu, *Student Member, IEEE*, and Jian Song, *Fellow, IEEE*

***Abstract*—The clipping noise, which could cause out-of-band radiation and increase bit error rate in orthogonal frequency division multiplexing systems, is necessary to be mitigated. This paper proposes a clipping noise elimination scheme based on compressed sensing (CS) with partially aware support. Specifically, the clipping noise is reconstructed from the selected reliable observations in the frequency domain with the proposed partially aware support sparsity adaptive matching pursuit algorithm. The proposed method outperforms the conventional clipping noise elimination approaches in the aspects of both the accuracy and robustness by exploiting the partially aware support, which is verified by both theoretical analysis and numerical simulation.**

***Index Terms*—Clipping noise, orthogonal frequency division multiplexing (OFDM), compressed sensing (CS), partially aware support.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM), as an attractive modulation scheme, has been widely adopted in wireless communication [1], optical communication [2], and broadcasting systems due to its high spectrum efficiency, robustness against frequency selective fading, and capability to support high-rate data transmission. The techniques of synchronization [3], timing correction [4], and carrier frequency offset estimation [5] for OFDM receiver have been extensively researched. However, one of the major drawbacks of OFDM modulation is the relatively large peak-to-average power ratio (PAPR), which tends to impair the power efficiency of the radio frequency amplifier.

Numerous methods have been proposed to deal with the PAPR problem in OFDM signals [6], [7], including clipping and filtering [8], block coding [9], tone reservation [10], tone injection [10], and active constellation extension (ACE) [11]. These techniques achieve the PAPR reduction at expense of the transmit signal power increase, bit error rate growth, data rate loss, computational complexity enhancement, and so on.

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The authors are with the Department of Electronic Engineering, Research Institute Information Technology, Tsinghua National Laboratory for Information Science and Technology, Tsinghua University, Beijing 100084, China (e-mail: fangyang@tsinghua.edu.cn).

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Besides these PAPR reduction methods mentioned above, the amplitude clipping is one of the simplest ways to reduce the PAPR, which directly limits the OFDM signals to a predetermined range [12], and then results in signal distortion such as in-band distortion and out-of-band radiation [13]. Moreover, the distortion caused by the clipping operation can be treated as another kind of noise, whose power spectrum falls both in-band and out-of-band [14]. However, the in-band distortion cannot be simply reduced by filtering thus leading to a performance degradation, while the out-of-band radiation decreases the spectrum efficiency and brings more severe interference to the adjacent channel.

There are a few techniques proposed to mitigate the harmful impacts of the clipping noise. In [15], a decision-aided reconstruction method is proposed. Based on the fact that the impact of the clipping noise is mitigated when decisions are made in the frequency domain, the signal is recovered from the harmful impacts of the clipping noise in the time domain. With extra fast Fourier transform (FFT) operations for each iteration at the receiver, this method is not computationally efficient in practice. Another way to compensate for the performance degradation from the clipping process is using the least square method for oversampled signal reconstruction in oversampled systems [16]. The clipped samples are considered as lost samples, and reconstructed based on the other samples in the oversampled signals. In [17], an iterative receiver is proposed to estimate and mitigate the clipping noise. Because of the fact that the clipping noise is generated by a known process, it can be recovered at the receiver using the iterative maximum likelihood (ML) estimation.

Recently, the theory and algorithms of compressed sensing (CS) [18]–[20], as an important breakthrough in the field of sparse signal processing [21]–[25], could be applied to the clipping noise estimation due to the time-domain sparse property of the clipping noise. According to the CS theory, a sparse signal can be reconstructed accurately from the compressed observations [26]. The CS-based clipping noise mitigation method is firstly investigated in [27], where the reserved subcarriers are utilized to reduce the peak and estimate the clipping noise. However, this scheme would cause the loss of spectrum efficiency because of the reserved tones. Considering the reliability of observations, an enhanced clipping noise cancellation method based on CS is introduced in [28]. This scheme does not need reserved tones, but selectively adopts the data tones as observations, which are less

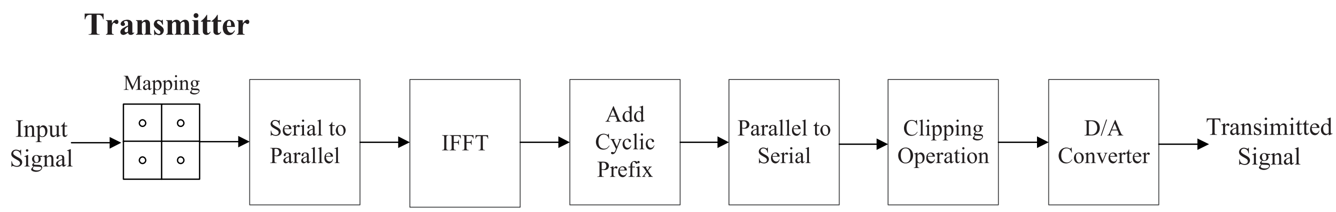

Fig. 1. The transmitter block diagram of the OFDM systems.

contaminated by the channel noise. In this way, reliable observations can be utilized without suffering data rate loss and the clipping noise is reconstructed by CS methods.

In this paper, a CS-based approach for OFDM systems with the aid of the partially aware support is investigated to eliminate the clipping noise exactly. Since the amplitudes of the distorted received OFDM samples, which have been clipped at the transmitter, are normally much higher than that of the other ordinary samples in the time domain, it is feasible to obtain a coarse estimation of the clipping noise locations as the partially aware support. By exploiting the partially aware support, the classical CS greedy algorithm, such as sparsity adaptive matching pursuit (SAMP) [29], is significantly improved. Compared with the conventional schemes, the accuracy and robustness of the proposed scheme are ensured and the computational complexity is also reduced.

Compared to the conventional methods that have different drawbacks and limitations, the CS method using partially aware support SAMP (PAS-SAMP) algorithm is capable of accurately reconstructing the exact clipping noise and removing it from the received signal, thus applicable to various channel with low complexity. Moreover, without requiring additional reserved tones, existing data tones are exploited with high spectrum efficiency, which can be applied to various communications systems. Therefore, the proposed method is not only effective in broadcasting systems, but can also adapt to other environments contaminated by the clipping noise.

The rest of this paper is organized as follows. Section II presents the OFDM system model including both the transmitter and the proposed CS-based receiver, and the channel model for OFDM systems is also described. Then, the proposed PAS-SAMP algorithm as well as the performance analysis are provided in Section III. Section IV demonstrates simulation results to validate the proposed approach. Finally, conclusions are drawn in Section V.

Notation: Matrices and vectors are denoted by boldface letters. $\|\cdot\|_r$ represents the ℓ_r norm operation; $(\cdot)^H$ and $(\cdot)^\dagger$ denote the conjugate transpose and pseudo-inversion operators, respectively; $\mathbf{v}|_\Pi$ and $\max(\mathbf{v}, T)$ denote the entries of the vector $\mathbf{v}$ in the set of Π and the indices of the largest T entries of the vector $\mathbf{v}$, respectively. $\mathbf{A}_\Pi$ and Π^c represent the sub-matrix comprised of the Π columns of the matrix $\mathbf{A}$ and the complementary set of Π, respectively. $|\Pi|$ denotes the cardinality of the set Π.

II. System Model

A. OFDM System Transmitter

As shown in Fig. 1, at the transmitter of the OFDM systems, after forward error coding and interleaving, the input signal is modulated by phase shift keying (PSK) or quadrature amplitude modulation (QAM), and can be represented as $\mathbf{X} = [X_0, X_1, X_2, \ldots, X_{N-1}]$ in the frequency domain, where X_k $(0 \le k < N)$ represents the complex data of the k-th subcarrier, and N is the number of subcarriers.

Then, after passing through the inverse fast Fourier transform (IFFT) module, the OFDM signal vector $\mathbf{x}$ in the time domain is given by

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \exp\left(\frac{j2\pi kn}{N}\right),\ 0 \le n < N. \tag{1}$$

The PAPR of the discrete time domain signal is defined as the ratio of the maximum peak power divided by the average power of the OFDM signal, and is represented as

$$\mathrm{PAPR}(\mathbf{x}) = \frac{\max_{0\le n<N}|x_n|^2}{\mathrm{E}\left\{|x_n|^2\right\}}, \tag{2}$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operation. From the central limit theorem, the real and imaginary parts of the time domain signal samples follow Gaussian distributions [6], thus the probability that the PAPR ξ of the Nyquist-rate sampled OFDM signals exceeds a given threshold ξ_0 is given by

$$\Pr(\xi > \xi_0) = 1 - (1 - \exp(-\xi_0))^N. \tag{3}$$

After a cyclic prefix (CP) is added, the clipping operation is performed on the OFDM signal in order to ensure the PAPR reduction. Therefore, the time-domain clipped signal can be represented as

$$\bar{x}_n = \begin{cases} x_n, & |x_n| \le A_{th}, \\ A_{th}e^{j\phi(x_n)}, & |x_n| > A_{th}, \end{cases} \tag{4}$$

where A_{th} is the clipping threshold, and $\phi(x_n)$ represents the phase of x_n. As a result, the clipped signal $\bar{x}_n$ in the time domain could be treated as the sum of original OFDM signal x_n and the clipping noise c_n, i.e.,

$$\bar{x}_n = x_n + c_n, \quad 0 \le n < N. \tag{5}$$

Hence, the expression of the clipped signal, the OFDM signal, and the clipping noise in the frequency domain is obtained as

$$\bar{X}_k = X_k + C_k, \quad 0 \le k < N, \tag{6}$$

where $\bar{X}_k$ and C_k represent the frequency-domain clipped signal and clipping noise for k-th subcarrier, respectively.

Finally, the discrete clipped signal is converted to the analogue signal using digital-to-analogue converters (DACs).

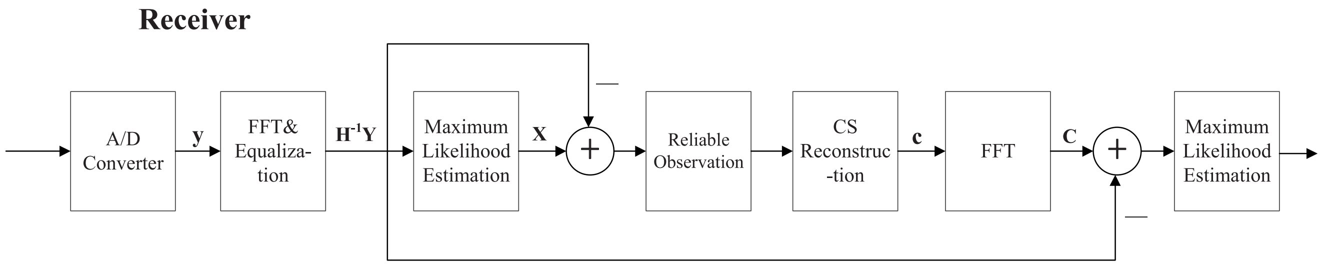

Fig. 2. The receiver block diagram of the proposed OFDM systems.

B. Channel Model for OFDM Systems

Using broadcasting systems as an example, we assume that the channel impulse response $\mathbf{h}$ comprises P resolvable propagation paths, which can be modeled as

$$h_n = \sum_{p=0}^{P-1} \alpha_p \delta\left[n - \tau_p\right], \quad 0 \le n < N, \tag{7}$$

where α_p and τ_p denote the normalized multipath gain and the sampled multipath delay of the p-th path, respectively. The sampled multipath delay set $\boldsymbol{\tau}$ is defined as

$$\boldsymbol{\tau} = \{\tau_0, \tau_1, \ldots, \tau_{P-1}\}, \tag{8}$$

where $0 \le \tau_0 < \tau_1 < \cdots < \tau_{P-1} \le N-1$ can be assumed without loss of generality.

Assuming that other interferences are eliminated, what arrives at the receiver is the convolution of the transmitted signal and the channel impulse response plus channel noise, which is denoted by $\mathbf{y}$ and given by

$$\mathbf{y} = \mathbf{h} \otimes \bar{\mathbf{x}} + \mathbf{z}, \tag{9}$$

where $\otimes$ is the convolution operator and $\mathbf{z}$ represents the channel noise, which is usually modeled as the additive white Gaussian noise (AWGN) in the time domain.

C. CS-Based OFDM System Receiver

1) Initial Estimation Without Clipping Noise Cancellation: As shown in Fig. 2, at the receiver, the signal passes through an analogue-to-digital converter (ADC) firstly, and then the CP is removed. After the FFT process, the received symbol Y_k in the frequency domain can be represented as

$$Y_k = H_k \cdot \bar{X}_k + Z_k, \quad 0 \le k < N, \tag{10}$$

where Z_k and H_k denote the frequency-domain AWGN and channel frequency response (CFR), respectively.

After zero-forcing channel equalization, a ML estimator will be imposed on the received symbol Y_k, and then the initial decision $\hat{X}_k$ is obtained as

$$\hat{X}_\mathrm{k} = \arg\min \left| {H_k}^{-1} Y_k - s \right|, \quad s \in \mathcal{X}, \tag{11}$$

where $\mathcal{X}$ denotes the presupposed constellation set associated with the adopted modulation approach.

2) Clipping Noise Reconstruction Based on CS: In the CS problem, the measurement vector is necessary to reconstruct the unknown sparse vector. In order to obtain the measurement vector of the clipping noise, the decision result $\hat{\mathbf{X}}$ in (11) is subtracted from (10) according to (6), i.e.,

$$\begin{aligned} \mathbf{H}^{-1}\mathbf{Y} - \hat{\mathbf{X}} &= \mathbf{X} + \mathbf{C} + \mathbf{H}^{-1}\mathbf{Z} - \hat{\mathbf{X}} \\ &= \mathbf{C} + \left(\mathbf{X} - \hat{\mathbf{X}}\right) + \mathbf{H}^{-1}\mathbf{Z} \\ &= \mathbf{C} + \boldsymbol{\theta}, \end{aligned} \tag{12}$$

where $\boldsymbol{\theta} = (\mathbf{X} - \hat{\mathbf{X}}) + \mathbf{H}^{-1}\mathbf{Z}$ denotes the observation noise, which is the combined effect of the decision error and channel noise. Obviously, $\mathbf{H}^{-1}\mathbf{Y} - \hat{\mathbf{X}}$ is equal to the sum of the clipping noise $\mathbf{C}$ and the observation noise $\boldsymbol{\theta}$.

Since a number of subcarriers are severely interfered by the channel fading or decision error, some reliable subcarriers among the entire tones should be selected to reduce the observation noise and improve the recovery performance by the selection matrix $\mathbf{S}$ with the size $M \times N$. The selection matrix $\mathbf{S}$ is consisting of M rows of the $N \times N$ identity matrix $\mathbf{I}$, which improves the reliability of observations. Therefore, the measurement vector $\tilde{\mathbf{Y}}$ after the reliable subcarriers selection can be given by

$$\begin{aligned} \tilde{\mathbf{Y}} &= \mathbf{S}\left(\mathbf{H}^{-1}\mathbf{Y} - \hat{\mathbf{X}}\right) = \mathbf{S}(\mathbf{C} + \boldsymbol{\theta}) \\ &= \mathbf{SFc} + \mathbf{S}\boldsymbol{\theta} \\ &= \boldsymbol{\Phi}\mathbf{c} + \boldsymbol{\eta}, \end{aligned} \tag{13}$$

where $\mathbf{F}$ is the unitary fast Fourier transform (FFT) matrix with the size $N \times N$, and $\boldsymbol{\Phi} = \mathbf{SF}$ can be considered as the $M \times N$ sensing matrix, which shows a good restricted isometry property (RIP) [30] and will be discussed in detail at Section III-E. The clipping noise $\mathbf{c}$ and $\boldsymbol{\eta} = \mathbf{S}\boldsymbol{\theta}$ represent the sparse signal vector and observation noise vector, respectively, according to the CS theory. By using a CS recovery algorithm such as the proposed PAS-SAMP, we can recover $\mathbf{c}$ as $\hat{\mathbf{c}}$.

In the proposed method, the reliable subcarriers, whose power of the additional observation noise is lower than the average power of the clipping noise, are selected [28]. Therefore, the reliable subcarrier set can be represented as

$$\mathcal{K} = \left\{k \middle| \; |\theta_k|^2 < \mathrm{E}\left\{|\mathrm{C_k}|^2\right\}\right\}. \tag{14}$$

3) OFDM Symbols Reestimation: Afterwards, the estimated clipping noise vector $\hat{\mathbf{C}}$ is subtracted from the frequency-domain symbol $\mathbf{Y}$ to acquire the final decision $\tilde{X}_k$, which is represented as

$$\tilde{X}_k = \arg\min\left|{H_k}^{-1}Y_k - \hat{C}_k - s\right|, \qquad s \in \mathcal{X}. \tag{15}$$

After clipping noise cancellation is performed in the time domain, the final estimated signal can be processed by the successive demapping and decoding modules at the receiver.

III. Proposed Method Based on Compressed Sensing With Partially Aware Support

A. CS Recovery Algorithm

It is feasible to recover the clipping noise in the presence of the observation noise $\boldsymbol{\eta}$ based on the CS theory from (13). Proofs have been given that solving the CS measurement in (13) is equivalent to solving the convex optimization problem [18],

$$\min\|\mathbf{c}\|_1 \quad \text{s.t.}\left\|\tilde{\mathbf{Y}} - \boldsymbol{\Phi}\mathbf{c}\right\|_2 \le \boldsymbol{\varepsilon} \tag{16}$$

where $\boldsymbol{\varepsilon}$ is the bound of the observation noise $\boldsymbol{\eta}$.

After convex relaxation, the l_1 norm minimization problem given by (16) can be efficiently solved through greedy algorithms to recover the unknown clipping noise vector $\mathbf{c}$ accurately. Among the existing CS greedy algorithms, the SAMP algorithm does not require knowledge of the sparsity level, whereas the others such as orthogonal matching pursuit (OMP) and subspace pursuit (SP) need to know the sparsity information. In practical systems at the receiver, the sparsity level of the clipping noise is unknown and can be variant. Hence, we adopt the SAMP algorithm in our scenarios. The main mechanism of the SAMP is by iteratively updating the testing sparsity level K_T, the sparse vector is approached with the decrease of the residual norm. At a certain testing sparsity level, a preliminary test list is first derived by the projection of the residue onto the whole dictionary. Afterwards, through the projection of the measurement vector onto the plane spanned by the temporary list, a candidate support is acquired by choosing the maximal K_T projections. If the residual norm decreases, it means that the iteration should be continued within the same stage, whereas the stage should be switched by increasing the testing sparsity level K_T by a step size Δs if the residual norm increases. When the halting condition is met, the exact recovery is achieved.

Moreover, the classical SAMP algorithm could be improved with the aid of the partially aware support, which can be obtained from the receiver directly. The proposed PAS-SAMP algorithm will enhance the accuracy and robustness of the clipping noise reconstruction performance, which will be described in detail in the following content.

B. Acquisition of Partially Aware Support

Since the amplitude of the clipping signal is usually much higher than that of the other unclipped signal in the time domain at the transmitter, the coarse estimation of the partial support $\Pi^{(0)}$ at the receiver could be effectively achieved.

For the multipath channel, in consideration of the channel impulse response, the partially aware support could be obtained from the initial decision $\hat{\mathbf{x}}$ in the time domain, instead of the received time-domain signal $\mathbf{y}$, due to the convolution of the signal propagation under the multipath channel. The partially aware support $\Pi^{(0)}$ should include the samples whose powers are larger than the given threshold λ_t in the time domain, and is given by

$$\Pi^{(0)} = \left\{n\middle|\ \left|\hat{x}_n\right|^2 > \lambda_t, \quad 0 \le n < N\right\}, \tag{17}$$

where $\hat{x}_n$ is the initial decision in the time domain, and the power threshold λ_t is represented as

$$\lambda_t = \alpha\sum_{n=0}^{N-1}\left|\hat{x}_n\right|^2, \tag{18}$$

where α is a configurable coefficient according to different conditions, which guarantees the accuracy of the partially aware support in the time domain.

Particularly, for the channel impulse response, which has a main path with much larger amplitude than those of other resolvable propagation paths, the partially aware support could be obtained from the received time-domain signal $\mathbf{y}$ for simplicity.

Algorithm 1 PAS-SAMP: The Partially Aware Support Sparsity Adaptive Matching Pursuit for Clipping Noise Estimation

Input:

1) Measurement vector $\tilde{\mathbf{Y}}$
2) Sensing matrix $\boldsymbol{\Phi}$
3) The partially aware support $\Pi^{(0)}$
4) Step size Δs.

Initialization:

$K_0 = |\Pi^{(0)}|$

$\mathbf{c}^{(0)}\big|_{\Pi^{(0)}} = \boldsymbol{\Phi}^{\dagger}_{\Pi^{(0)}}\tilde{\mathbf{Y}}$

$\mathbf{r}^{(0)} = \tilde{\mathbf{Y}} - \boldsymbol{\Phi}\mathbf{c}^{(0)}$

$K_T = K_0 + \Delta s; \quad k = 1; \quad i = 1$

Iterations:

repeat

  $P_k = \max(\boldsymbol{\Phi}^H\mathbf{r}^{(k-1)}, K_T - K_0)$ {Preliminary test}

  $C_k = \Pi^{(k-1)} \cup P_k$ {Make candidate list}

  $\Pi_t = \max(\boldsymbol{\Phi}^{\dagger}_{C_k}\tilde{\mathbf{Y}}, K_T)$ {Temporary final list}

  $\mathbf{c}^{(k)}\big|_{\Pi_t} = \boldsymbol{\Phi}^{\dagger}_{\Pi_t}\tilde{\mathbf{Y}},\ \mathbf{c}^{(k)}\big|_{\Pi_t^c} = \mathbf{0}$

  $\mathbf{r}^{(k)} = \tilde{\mathbf{Y}} - \boldsymbol{\Phi}_{\Pi_t}\boldsymbol{\Phi}^{\dagger}_{\Pi_t}\tilde{\mathbf{Y}}$ {Residue computation}

  if $\left\|\mathbf{r}^{(k)}\right\|_2 < \left\|\mathbf{r}^{(k-1)}\right\|_2$ **then**

    $\Pi^{(k)} = \Pi_t,$

    $k = k + 1$ {Same stage, next iteration}

  else

    $k = 1, i = i + 1,$

    $K_T = K_0 + i \times \Delta s$ {Stage switching}

  end if

until $\left\|\mathbf{r}^{(k)}\right\|_2 < \boldsymbol{\varepsilon}$

Output:

Estimated clipping noise vector $\mathbf{c}$, s.t.
$\mathbf{c}|_{\Pi_i} = \boldsymbol{\Phi}^{\dagger}_{\Pi_i}\tilde{\mathbf{Y}},\ \mathbf{c}|_{\Pi_i^c} = \mathbf{0}$

C. The Proposed PAS-SAMP Algorithm

Algorithm 1 illustrates the proposed PAS-SAMP algorithm for the clipping noise estimation in the pseudo-code format.

In detail, $\tilde{\mathbf{Y}}$, $\mathbf{\Phi}$, and $\Pi^{(0)}$ are the input variables as the measurement vector, the sensing matrix, and the initial support, respectively. The iterative step size Δs, which is a configurable parameter for various scenarios, should be a tradeoff between the convergence rate and accuracy. The testing sparsity level K_T for the current stage is changed by $K_T = K_0 + i \times \Delta s$ during the iterations. The output is the final recovered unknown sparse vector.

The partially aware support $\Pi^{(0)}$ is exploited to initialize the support set, which could reduce the complexity of the total iterations compared with that of the SAMP. During the iteration process, the partially aware support is also made good use of to improve the accuracy of the temporary support estimation in each iteration, and to reduce the computational complexity.

Compared with the conventional SAMP algorithm, the PAS-SAMP has superior accuracy and adaptivity. On the one hand, the preliminary initialization in the proposed PAS-SAMP is more accurate than the trivial initialization in the conventional SAMP, meanwhile it is possible to adopt smaller step size Δs in the PAS-SAMP due to the fact that the test sparsity level is altered to $K_T = K_0 + i \times \Delta s$ compared with $K_T = i \times \Delta s$ in the conventional SAMP algorithm, which leads to more accurate estimation of the actual sparsity level K than that in the SAMP. On the other hand, since in different channel conditions, the initial support of the PAS-SAMP varies accordingly, and the contributions of the partially aware support will significantly facilitate the accurate clipping noise recovery. Therefore, the proposed PAS-SAMP algorithm is very suitable to the variant sparsity level K.

D. Complexity Analysis

The computational complexity of the proposed clipping noise cancellation scheme includes the following parts:

- In the first step, the complexity of the CS measurement vector acquisition is $\mathcal{O}(N)$, which is quite low.
- In the second step, since the complexity of FFT is $\mathcal{O}(N\log_2(N))$, the complexity of the partially aware support acquisition is $\mathcal{O}(N\log_2(N))$.
- In the third step of the PAS-SAMP, which contributes the major complexity, for each iteration, the complexity consists of two parts: the inner product between the sensing matrix $\mathbf{\Phi}$ and the residue $\mathbf{r}$ has complexity of $\mathcal{O}(MN)$; the equivalent least-squares (LS) problem $\mathbf{c}^{(k)}|_{\Pi_t} = \mathbf{\Phi}^{\dagger}_{\Pi_t}\tilde{\mathbf{Y}}$ requires the complexity of $\mathcal{O}(MK)$ when the Gram-Schmidt algorithm is used since the LS problem only relates to the sub-matrix $\mathbf{\Phi}^{\dagger}_{\Pi_t}$ that is not larger than $M \times K$. The average total number of iterations is reduced from K in the SAMP to $K - K_0$ in the PAS-SAMP, so the total complexity of the PAS-SAMP is in the order of $\mathcal{O}((K-K_0)M(N+K))$.
- The total complexity of the proposed clipping noise elimination approach is in the order of $\mathcal{O}((K-K_0)M(N+K) + N\log_2(N))$.

Compared with the conventional SAMP method with an empty initial support, the proposed PAS-SAMP utilizes $\Pi^{(0)}$, which is achieved by the partially aware support acquisition, as the initial support resulting in faster convergence rate.

TABLE I
THE MULTIPATH CHANNEL PARAMETERS FOR SIMULATIONS

Path Index	ITU V-B	
	Delay (μs)	Gain (dB)
1	0.00	-2.5
2	0.30	0.0
3	8.90	-12.8
4	12.90	-10.0
5	17.10	-25.2
6	20.00	-16.0

Moreover, since the initialization of the testing sparsity level is set to $K_T = K_0 + \Delta s$ instead of $K_T = \Delta s$ in the traditional SAMP approach, the average number of the iterations could be decreased. Therefore, the computational complexity of the PAS-SAMP algorithm is reduced by a factor of K_0/K compared to that of the conventional SAMP method. Meanwhile, the convergence rate of the iterations is also improved since the testing sparsity level starts much closer to the actual one.

E. Restricted Isometry Property Analysis

In CS, it is known that if a matrix $\mathbf{\Phi} \in \mathbb{C}^{M \times N}$ satisfies the RIP, it means for all K-sparse vector $\mathbf{x}$,

$$(1-\delta)\|\mathbf{x}\|_2^2 \leq \|\mathbf{\Phi x}\|_2^2 \leq (1+\delta)\|\mathbf{x}\|_2^2, \tag{19}$$

where δ denotes the RIP constant. Since the RIP is a sufficient condition that guarantees the correctness of recovery, many researches have been conducted to search the RIP matrices. Fortunately, the partial Fourier matrix has been proved to be a kind of RIP matrices [19], [31]. Candes and Tao [19] have proved that a matrix comprised of random rows from the Fourier matrix satisfies RIP with positive probability as long as the number of rows reaches $M = \mathcal{O}(K\log N)$.

On the other hand, we also verified through numerical analysis that the sensing matrix $\mathbf{\Phi} = \mathbf{SF}$ satisfies the RIP property. The RIP constant δ is required to be less than 0.41 to accurately recover the K-sparse signal in the presence of background noise [32]. Based on the numerical calculation, the RIP constant of the proposed observation matrix satisfies $\delta < 0.382$ for the sparse level $K = 10$. This implies that the partial Fourier matrix successfully meets the RIP requirements.

Furthermore, in our proposed scheme, the sparse selection matrix $\mathbf{\Phi}$ could select M rows on purpose, which is closer to the strict orthogonal isometric property with less observation noise, to guarantee the sensing matrix satisfying RIP condition.

IV. SIMULATION RESULTS

In this section, extensive simulations are carried out to evaluate the performance of the proposed CS-based clipping noise elimination with the aid of partially aware support for OFDM systems. The simulation parameters are configured according to the broadcasting system with the signal bandwidth of 7.56 MHz located at the central radio frequency of 634 MHz. The modulation scheme of 16-QAM and 64-QAM are adopted. Meanwhile, the typical six-tap International

TABLE II
AVERAGE NUMBER OF ITERATIONS FOR CLIPPING NOISE ESTIMATION

Sparsity Level	PAS-SAMP	SAMP
5	2.34	4.32
10	5.28	9.51
15	7.12	14.23
20	9.72	19.14

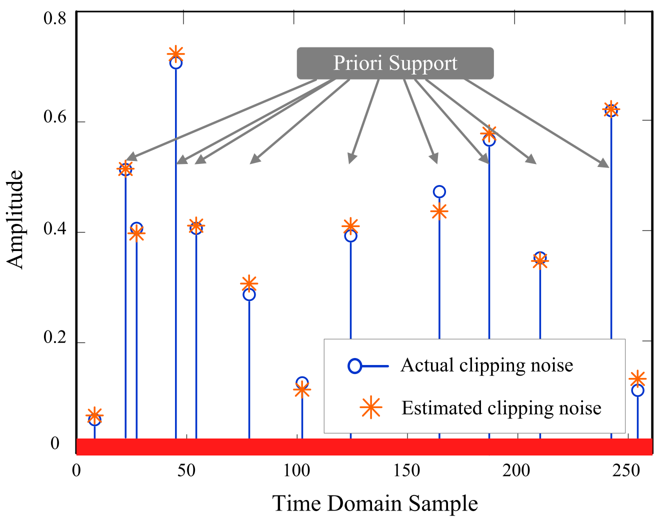

Fig. 3. Clipping noise reconstruction visualization using the proposed PAS-SAMP method.

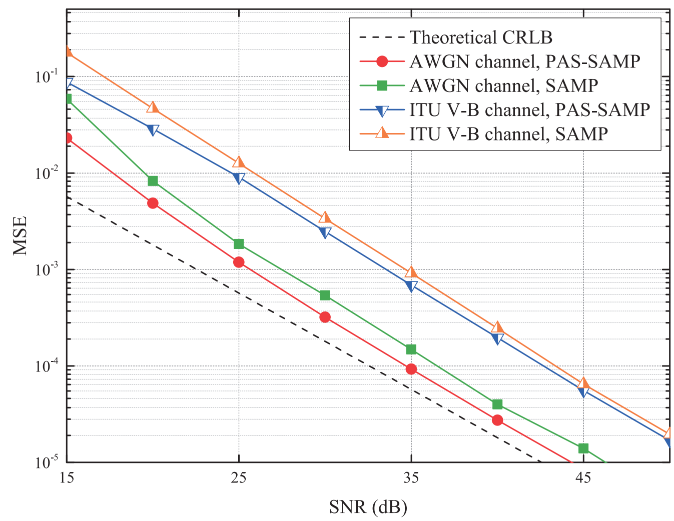

Fig. 4. MSE performance comparison with the CS-based PAS-SAMP and SAMP algorithms under both AWGN and ITU V-B multipath channels.

Telecommunications Union Vehicular B (ITU V-B) channel [33] is adopted as the multipath fading channel model for the performance evaluation. The multipath channel parameters for simulations are listed in Table I.

To validate the complexity analysis in Section III-D, the average numbers of iterations for the clipping noise estimation using the proposed PAS-SAMP and conventional SAMP algorithms are summarized in Table II. The average number of iterations is obtained from 10^4 times numerical simulations in the same condition for both PAS-SAMP and SAMP algorithms. It is noted that the number of iterations for the PAS-SAMP is significantly decreased compared with that of the SAMP method, which demonstrates that the proposed PAS-SAMP can further reduce the complexity of the existing greedy algorithm SAMP by exploiting the partially aware support.

Fig. 3 depicts a general visualization of the proposed scheme for the partially aware support aided clipping noise reconstruction, with the parameters of 16-QAM constellations, sub-carrier number $N = 256$, and the clipping threshold $A_{th} = 1.6$. The partially aware support is firstly obtained using the power threshold $\lambda_t = 1.5$ in (17). From the reliable observations in the frequency domain, the clipping noise is estimated using the proposed PAS-SAMP algorithm. It is observed from Fig. 3 that the final clipping noise estimation matches the actual clipping noise well.

The mean square error (MSE) performance comparisons of the proposed scheme with the CS-based PAS-SAMP and SAMP algorithms are shown in Fig. 4. The performance of the clipping noise reconstruction under both AWGN and ITU V-B channels are depicted with the parameters of 16-QAM constellations, sub-carrier number $N = 256$, and the clipping threshold $A_{th} = 1.6$, where the sparsity level K is approximately equal to 20 through vast numerical simulation. The theoretical Cramer-Rao lower bound (CRLB) $\sigma^2(K/M)$ is also included for comparison [34]. It is noted that the proposed CS method with the PAS-SAMP achieves a target MSE of 10^{-3} at the SNR of 27 dB and 33 dB under the AWGN and ITU V-B channels, respectively, both of which outperform the SAMP method by approximately 2 dB. Meanwhile, the MSE performance of the proposed PAS-SAMP method approaches the theoretical CRLB with the increase of the SNR. As a consequence, the MSE performance validates the high accuracy of the proposed PAS-SAMP method for the clipping noise reconstruction, which fully exploits the sparse characteristics of the clipping noise and partially aware support.

The recovery probabilities of the proposed CS methods for the PAS-SAMP and SAMP with the fixed SNR of 35 dB are shown in Fig. 5. The recovery probabilities for different modulation schemes under the AWGN channel are simulated with the sub-carrier number $N = 256$. The recovery probability is the ratio of successful OFDM symbols estimation, which is defined as the MSE $< 10^{-2}$. It can be seen from Fig. 5 that the recovery probability rises up along with the increase of the clipping threshold. This is due to the fact that the sparsity level of the clipping noise is reduced when lower clipping threshold is adopted according to the distribution of the time-domain OFDM signal, which results in superior recovery performance. The proposed PAS-SAMP method reaches a successful recovery probability of 0.9 with the clipping threshold $A_{th} \geq 1.2$ for 16-QAM under the AWGN channel. This indicates that with the aid of the partially aware support, the PAS-SAMP method can correctly recover the clipping noise only using

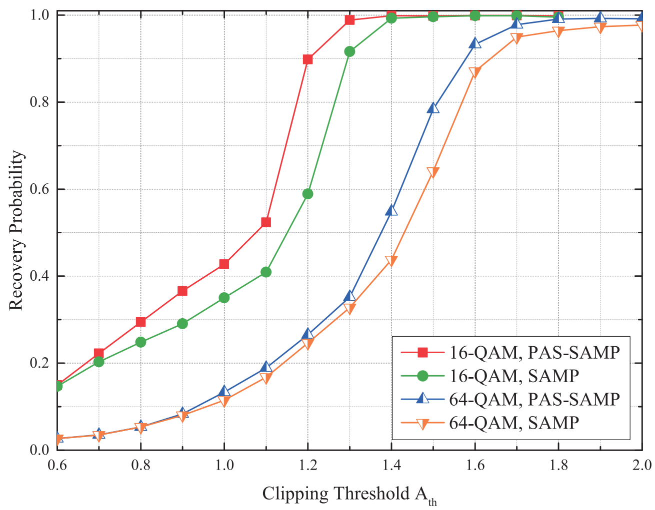

Fig. 5. The recovery probability of OFDM symbols using the PAS-SAMP and SAMP algorithms under the AWGN channel.

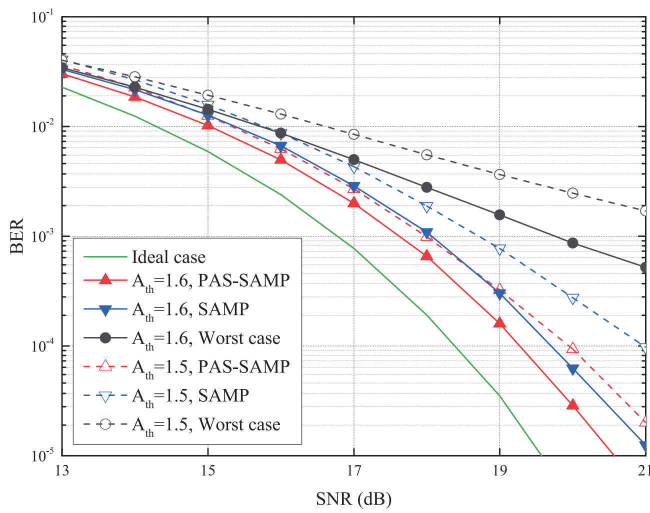

Fig. 6. The BER performance versus SNR under the AWGN channel when $N = 256$, 16-QAM are used.

the selected reliable observations in the frequency domain. It is also noted from the gap between the PAS-SAMP and SAMP curves that the proposed PAS-SAMP method can accurately recover the clipping noise at larger sparsity levels by exploiting the partially aware support.

The bit error rate (BER) of the proposed system based on the PAS-SAMP algorithm is computed, while the conventional CS-based SAMP algorithm is presented for comparison. The worst case ignoring clipping noise and the ideal case without clipping noise are also depicted as benchmarks. Specifically, the BER performances of the proposed clipping noise cancellation scheme under both AWGN and ITU V-B channels with 16-QAM are shown in Fig. 6 and Fig. 7, respectively. The OFDM sub-carrier number is 256 and the clipping thresholds A_{th} are set to 1.5 and 1.6 for comparison. As shown in Fig. 6, at the target BER of 10^{-3}, the proposed scheme outperforms the conventional SAMP method by approximately 0.4 dB and 0.7 dB for $A_{th} = 1.6$ and $A_{th} = 1.5$, respectively. While in Fig. 7, the improvement is about 0.3 dB for $A_{th} = 1.6$ and 1.0 dB

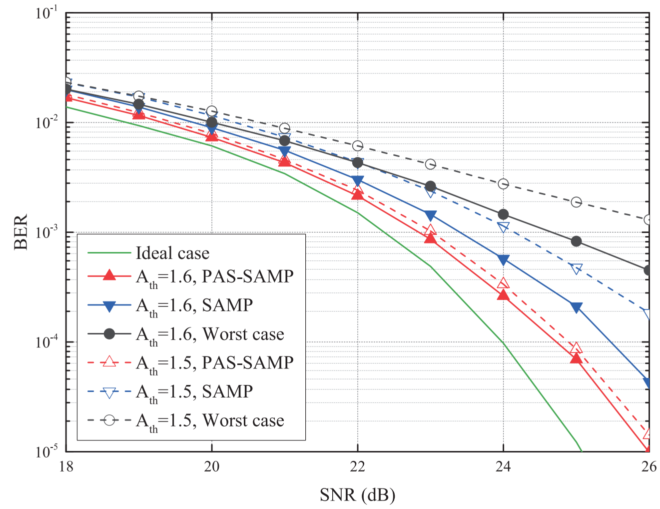

Fig. 7. The BER performance versus SNR under the ITU V-B channel when $N = 256$, 16-QAM are used.

for $A_{th} = 1.5$, respectively. It is clear from the curves that the proposed CS method with partially aware support could facilitate the accurate clipping noise recovery compared with the traditional CS method especially when the sparsity level becomes large. Meanwhile, as shown in Fig. 6 and Fig. 7, due to the increase of the sparsity level with lower clipping threshold, the system performance using the PAS-SAMP algorithm with $A_{th} = 1.5$ is degraded by 0.3 dB and 0.2 dB at the target BER of 10^{-3} compared with that when $A_{th} = 1.6$ is adopted under AWGN and ITU-VB channels, respectively. Therefore, the selection of the clipping threshold should be a tradeoff between low PAPR and high system performance. Moreover, the gap between the proposed method and the ideal case is only about 0.7 dB in Fig. 6 and 0.5 dB in Fig. 7, respectively, which validates the accuracy and effectiveness of the proposed recovery and elimination method. Furthermore, while the performance of the SAMP-based method degrades due to the channel fading, the proposed PAS-SAMP is hardly influenced by the multipath channel with the partially aware support, and is insensitive to the channel conditions.

V. Conclusion

In this paper, an improved method based on the partially aware support aided CS is proposed to accurately reconstruct and eliminate the clipping noise for OFDM systems. The proposed scheme does not need reserved pilots or tones, and instead, the selected reliable subcarriers among data tones in the frequency domain are adopted. By exploiting the partially aware support, which is a coarse estimation of the clipping noise location in the time domain, the proposed CS reconstruction algorithm is greatly improved, and the accuracy of the PAS-SAMP is further optimized by a threshold-based support adjustment method. With the aid of the partially aware support, the proposed PAS-SAMP method significantly improves both the accuracy and robustness of the clipping noise elimination with lower complexity compared with the conventional counterparts, which is validated by extensive simulations.

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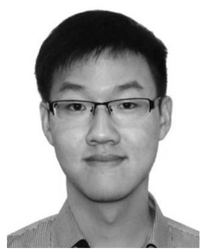
Fang Yang (M'11–SM'13) received the B.S.E. and Ph.D. degrees from the Department of Electronic Engineering, Tsinghua University, Beijing China, in 2005 and 2009, respectively. He is currently an Associate Professor with the DTV Technology Research and Development Center, Tsinghua University. His research interests lie in the field of digital television terrestrial broadcasting, power line communication, and visible light communication along with the wireless communication.



Junnan Gao (S'16) received the B.S.E. degree in electronic engineering from the Department of Electronic Engineering, Tsinghua University, Beijing, China. He is currently pursuing the master's degree in electronic engineering from the DTV Technology Research and Development Center, Department of Electronic Engineering, Tsinghua University. His research interests include visible light communications and wireless communication system.

Sicong Liu (S'15) received the B.S.E. degree in electronic engineering from the Department of Electronic Engineering, Tsinghua University, Beijing, China. He is currently pursuing the Ph.D. degree in electronic engineering from the DTV Technology Research and Development Center, Department of Electronic Engineering, Tsinghua University. His research interests include power line communications, broadband transmission techniques, and interference mitigation.

Jian Song (M'06–SM'10–F'16) received the B.Eng. and Ph.D. degrees in electrical engineering from Tsinghua University, Beijing, China, in 1990 and 1995, respectively. He worked with Tsinghua University upon his graduation. He was with the Chinese University of Hong Kong and the University of Waterloo, Canada, in 1996 and 1997, respectively. He has been with Hughes Network Systems, USA, for seven years before joining the faculty team in Tsinghua in 2005, as a Professor. He is currently the Director of Tsinghua's DTV Technology Research and Development Center. He has published over 200 peer-reviewed journal and conference papers. He has been working in quite different areas of fiber-optic, satellite, and wireless communications, as well as the power line communications. His current research interest is in the area of digital TV broadcasting. He holds two U.S. and over 40 Chinese patents. He is fellow of IET.