Priori Aided Compressed Sensing-Based Clipping Noise Cancellation for ACO-OFDM Systems

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***Abstract*— In this letter, the clipping noise is reconstructed from the selected reliable observations in the frequency domain based on the modified compressed sensing (CS) algorithm for the asymmetrically clipped optical orthogonal frequency division multiplexing modulated visible light communication system. With the aid of the priori information obtained from the received time-domain signals, the proposed priori aided sparsity adaptive matching pursuit method improves the accuracy and robustness of the recovery performance. Simulation results show that the proposed scheme outperforms conventional CS-based clipping noise cancellation counterparts.**

***Index Terms*— ACO-OFDM, clipping noise, compressed sensing.**

I. Introduction

With the dramatic development of light emitting diodes (LEDs) and highly sensitive photodiodes (PDs), visible light communication (VLC) has been emerging as a promising candidate for the future wireless communication [1]. As an alternative of the conventional orthogonal frequency division multiplexing (OFDM), the asymmetrically clipped optical OFDM (ACO-OFDM) with non-negative transmitted signal has been employed in VLC systems owing to the advantages in high spectrum efficiency and simple implementation [2].

Due to the nonlinear transfer characteristics of LEDs, the time-domain OFDM signals exceeding the LED dynamic region are clipped, which causes the clipping noise and deteriorates the performance of VLC systems [3]. Recently, the theory of compressed sensing (CS) [4], which is a breakthrough in the research of sparse signal processing [5], is introduced to estimate the clipping noise due to its time-domain sparsity. The first work of reconstructing the clipping noise using CS is introduced in [6], where the reserved tones in the frequency domain are observed to estimate the clipping noise. Another clipping noise cancellation scheme using CS for OFDM systems is put forward in [7], which uses reliable data tones with less contamination by the channel noise.

In this letter, an improved method based on the priori aided CS is proposed to accurately reconstruct the clipping noise for ACO-OFDM systems. Compared with the conventional OFDM systems, we make a modification to eliminate the correlations in the sensing matrix for ACO-OFDM systems. Moreover, the Hermitian symmetry of ACO-OFDM signals in the frequency domain is additionally utilized to obtain the reliable observations. Furthermore, the non-positive property of the time-domain clipping noise is adopted to reconstructed the unknown sparse vector. With the support of the priori information, we proposed the improved prior aided sparsity adaptive matching pursuit (PA-SAMP) algorithm based on the classical CS algorithm SAMP [8], which has a superior performance compared to conventional counterparts.

The remainder of this letter is organized as follow. Section II presents the system model of the ACO-OFDM system with the clipping noise. Then, the proposed clipping noise cancellation approach based on the PA-SAMP algorithm is introduced in Section III. Section IV demonstrates simulation results to validate the proposed approach. Finally, conclusions are drawn in Section V.

Notation: Matrices and vectors are denoted by boldface letters; $(\cdot)^\dagger$ and $(\cdot)^H$ denote the pseudo-inversion operation and conjugate transpose; $\|\cdot\|_r$ represents the ℓ_r norm operation; $|\Pi|$ denotes the cardinality of the set Π; $\mathbf{v}|_\Pi$ denotes the entries of the vector $\mathbf{v}$ in the set of Π; $\mathbf{A}_\Pi$ represents the sub-matrix comprised of the Π columns of the matrix $\mathbf{A}$; Π^c denotes the complementary set of Π; $\max(\mathbf{v}, T)$ denotes the indices of the T largest entries of the vector $\mathbf{v}$.

II. System Model With Clipping Noise

For the ACO-OFDM system, the Hermitian symmetry is satisfied to guarantee the real value requirement, while only the odd subcarriers are occupied and the asymmetrically clipped operation is adopted to ensure the non-negative signal. As shown in Fig. 1, the transmitted symbol $\mathbf{X} = [X_0, X_1, X_2, \cdots, X_m, \cdots, X_{N-1}]$, where $X_k = X^*_{N-k}$, $X_k = 0$ if k is even, and N is the number of OFDM subcarriers.

Then, the time-domain ACO-OFDM signal vector $\mathbf{x}$ is obtained by the inverse fast Fourier transform (IFFT) process. After an asymmetrically clipped operation is performed, the time-domain signal is non-negative, and can be represented as

$$x_{\mathrm{ACO},n} = \begin{cases} x_n, & x_n \geq 0, \\ 0, & x_n < 0. \end{cases} \tag{1}$$

It is known that there is an anti-symmetry property in the time domain [9], i.e., $x_n = -x_{n+N/2}$, where $0 \leq n < N/2$, thus leading to the distortion introduced in the asymmetrically

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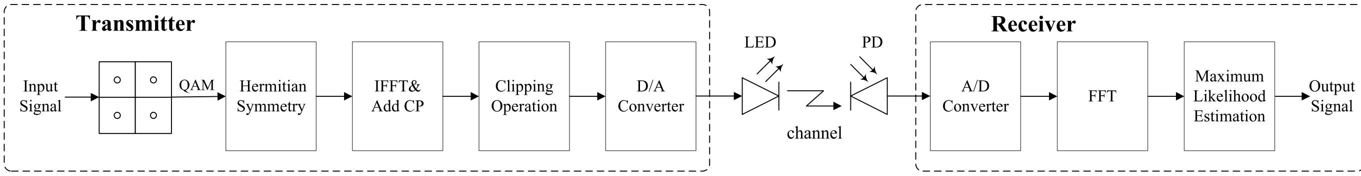

Fig. 1. The block diagram of the ACO-OFDM systems with the clipping noise.

clipped operation only falling on the even subcarriers [2], and the frequency-domain signal of $x_{\text{ACO},n}$ is given by

$$X_{\text{ACO},k} = X_k/2, \quad \text{if} \quad k \quad \text{is} \quad \text{odd}. \tag{2}$$

Due to the non-linear transfer characteristic of the LEDs, parts of the time-domain signals, whose amplitudes are beyond the dynamic range, are clipped, i.e.,

$$\bar{x}_{\text{ACO},n} = \begin{cases} x_{\text{ACO},n}, & x_{\text{ACO},n} \le A_{th}, \\ A_{th}, & x_{\text{ACO},n} > A_{th}, \end{cases} \tag{3}$$

where A_{th} is the clipping threshold.

The clipped signal $\bar{x}_{\text{ACO},n}$ can be considered as the sum of the original ACO-OFDM signal $x_{\text{ACO},n}$ and the clipping noise c_n. The expression of the clipped signal in the time and frequency domains can be presented as

$$\bar{x}_{\text{ACO},n} = x_{\text{ACO},n} + c_n, \quad 0 \le n < N. \tag{4}$$

$$\bar{X}_{\text{ACO},k} = X_{\text{ACO},k} + C_k, \quad \text{if} \quad k \quad \text{is} \quad \text{odd}, \tag{5}$$

where $\bar{X}_{\text{ACO},k}$ and C_k denote the corresponding frequency signals for the clipped signal and clipping noise at index k.

For the VLC communication system, the VLC channel can be modeled to consist of a line-of-sight (LOS) component and a diffuse or non-line-of-sight (NLOS) component. The channel impulse response (CIR) of directed light can be modeled by Dirac pulses whereas the diffuse portion can be represented by an integrating-sphere model [10]. The received symbol in the frequency domain Y_k can be represented as

$$Y_k = H_k \cdot \bar{X}_{\text{ACO},k} + Z_k, \quad \text{if } k \quad \text{is} \quad \text{odd}, \tag{6}$$

where H_k represents the channel frequency response (CFR), and Z_k denotes the additive white gaussian noise (AWGN) in the frequency domain.

III. Prior Aided Compressed Sensing Based Clipping Noise Reconstruction

A. Conventional Compressed Sensing Model

At the receiver, the received ACO-OFDM signal will go through a maximum likelihood (ML) estimator, and the estimation of transmitted symbol $\hat{X}_k$ is given by

$$\hat{X}_k = \arg\min |2{H_k}^{-1}Y_k - s|, \quad s \in \mathcal{X}, \tag{7}$$

where $\mathcal{X}$ is the signal constellation set corresponding to the modulation schemes.

By subtracting the estimation symbol $\hat{X}_k$ from the received symbol Y_k, the measurement vector used for the unknown sparse clipping noise reconstruction is presented as

$$\begin{aligned} {H_k}^{-1}Y_k - \frac{\hat{X}_k}{2} &= X_{\text{ACO},k} + C_k + {H_k}^{-1}Z_k - \frac{\hat{X}_k}{2} \\ &= \underbrace{C_k}_{\text{clipping noise}} + \underbrace{\theta_k}_{\text{observation noise}}, \end{aligned} \tag{8}$$

where k is odd and $\theta_k = X_{\text{ACO},k} - \hat{X}_k/2 + {H_k}^{-1}Z_k$ denotes the observation noise.

Due to the fact that some subcarriers are severely contaminated by the observation noise, an $M \times (N/2)$ selection matrix $\mathbf{S}$ is adopted to select the reliable tones from the odd subcarriers, which could improve the reconstruction accuracy. Therefore, the measurement vector $\tilde{\mathbf{Y}}$ can be given by

$$\begin{aligned} \tilde{\mathbf{Y}} &= \mathbf{S}(\mathbf{H}^{-1}\mathbf{Y} - \hat{\mathbf{X}}/2) = \mathbf{SC} + \mathbf{S}\boldsymbol{\theta} \\ &= \mathbf{SFc} + \mathbf{S}\boldsymbol{\theta} = \boldsymbol{\Phi}\mathbf{c} + \boldsymbol{\eta}, \end{aligned} \tag{9}$$

where $\mathbf{F}$ is an $(N/2) \times N$ partial discrete Fourier transform (DFT) matrix which is composed of odd rows of DFT matrix. $\boldsymbol{\Phi} = \mathbf{SF}$ can be considered as the $M \times N$ sensing matrix. The clipping noise $\mathbf{c}$ is the sparse signal vector in the time domain, and $\boldsymbol{\eta} = \mathbf{S}\boldsymbol{\theta}$ is the observation noise vector.

B. The Modification of CS Problem

Researches have shown that the restricted isometry property (RIP) is required for the sensing matrix [11]. However, the $(N/2) \times N$ partial DFT matrix $\mathbf{F}$ for ACO-OFDM systems is particular which would bring some troubles, i.e.,

$$F_{m,n+\frac{N}{2}} = e^{-j\frac{2\pi}{N}(2m+1)(n+\frac{N}{2})} = -e^{-j\frac{2\pi}{N}(2m+1)n} = -F_{m,n}, \tag{10}$$

where $F_{m,n}$ denotes the m-th row and n-th column element of the partial DFT matrix $\mathbf{F}$. Therefore, the n-th and $n + N/2$-th columns of the sensing matrix $\boldsymbol{\Phi}$, which is consist of odd rows of the DFT matrix, have the negative correlation, i.e.,

$$\Phi_{m,n} = -\Phi_{m,n+\frac{N}{2}}, \quad 0 \le m < M, \ 0 \le n < N, \tag{11}$$

where $\Phi_{m,n}$ represents the m-th row and n-th column element of the sensing matrix $\boldsymbol{\Phi}$. Consequently, the RIP is not satisfied, and the CS problem in the clipping noise cancellation for the ACO-OFDM system should be modified.

The sensing matrix $\boldsymbol{\Phi}$ can be represented as $[\mathbf{A}, -\mathbf{A}]$, where $\mathbf{A}$ is an $M \times (N/2)$ matrix. The clipping noise vector $\mathbf{c}$ can be represented as $[\mathbf{c_1}; \mathbf{c_2}]$, where $\mathbf{c_1}$ and $\mathbf{c_2}$ are $N/2 \times 1$ column vectors. Then, the measurement vector $\tilde{\mathbf{Y}}$ can be rewritten as

$$\tilde{\mathbf{Y}} = [\mathbf{A}, -\mathbf{A}] \cdot \begin{bmatrix} \mathbf{c_1} \\ \mathbf{c_2} \end{bmatrix} + \boldsymbol{\eta} = \mathbf{A}\tilde{\mathbf{c}} + \boldsymbol{\eta}, \tag{12}$$

where $\mathbf{A}$ and $\tilde{\mathbf{c}} = \mathbf{c_1} - \mathbf{c_2}$ are the new sensing matrix and sparse vector for the ACO-OFDM system, respectively. By using a CS recovery algorithm, $\tilde{\mathbf{c}}$ can be effectively recovered, and then $\mathbf{c_1}$ and $\mathbf{c_2}$ can be obtained from $\tilde{\mathbf{c}}$, which will be described in the subsection III-F.

C. Selection Criterion of Observations

The proposed criterion selects the reliable and odd subcarriers whose observation noise is lower than the average clipping noise power, meanwhile the Hermitian symmetry of the ACO-OFDM system should be also satisfied, which is given by

$$\mathcal{K} = \{k : (|\theta_k|^2 < E\{|C_k|^2\})\&(\hat{X}_k = \hat{X}^*_{N-k})\},\ k \text{ is odd}. \tag{13}$$

For stability of CS recovery algorithm, the cardinality $M = |\mathcal{K}|$ should be larger than a threshold $\mathcal{O}(K \cdot ln(N))$ for the sparse clipping noise recovery [8].

D. Acquisition of the Priori Information

Since the intensity of the received signal with the clipping noise is normally much higher than that of the other time domain signal components, it is feasible to obtain a coarse estimation of the partial support $\Pi^{(0)}$ at the receiver. In consideration of the LOS component is much larger than other NLOS components, the priori information could be obtained from the received ACO-OFDM signal y_n for simplicity. The time-domain samples whose powers exceed the given threshold λ_t are included in the partial support $\Pi^{(0)}$, which is presented as

$$\Pi^{(0)} = \{n|\ |y_n|^2 > \lambda_t,\ 0 \leq\ n < N\}. \tag{14}$$

where the power threshold λ_t is given by

$$\lambda_t = \alpha \sum_{n=0}^{N-1} |y_n|^2. \tag{15}$$

where α is a coefficient that can be configured large enough to ensure the accuracy of the time-domain partial support. In consequence of the modification of the CS problem in (12), the partial support need to be shifted correspondingly.

E. Priori Aided SAMP Algorithm

It has been proved that solving the CS problem in (12) is equivalent to solving the convex optimization problem [4], which can be effectively settled by CS greedy algorithms. Since the sparsity level of the clipping noise is variable and unknown for the receiver, the PA-SAMP algorithm is adopted for its adaptability to sparsity.

The pseudo-code of the PA-SAMP algorithm for the clipping noise reconstruction is summarized in Algorithm 1. Specifically, the inputs include the measurement vector $\tilde{\mathbf{Y}}$, the sensing matrix $\mathbf{\Phi}$, and the priori information support $\Pi^{(0)}$, as well as the iteration step size Δs. The output is the final recovered clipping noise vector.

Compared with the SAMP algorithm, the PA-SAMP algorithm uses the priori information $\Pi^{(0)}$ as the initial support instead of an empty set Ø in SAMP, which brings many advantages.

1) Complexity: The testing sparsity level T is initialized as $T \leftarrow K^{(0)} + \Delta s$ instead of $T \leftarrow \Delta s$ in the SAMP, so that the average number of total iterations is reduced from K in the SAMP to $K - K^{(0)}$ in the PA-SAMP, which reduces the complexity by a factor of $K^{(0)}/K$.

Algorithm 1 PA-SAMP: The Priori Aided Sparsity Adaptive Matching Pursuit for the Clipping Noise Reconstruction

Input:
1) The priori information support $\Pi^{(0)}$
2) Initial sparsity level $K^{(0)} = |\Pi^{(0)}|$
3) Measurement vector $\tilde{\mathbf{Y}}$
4) Sensing matrix $\mathbf{\Phi}$
5) Step size Δs.

Initialization:
1: $\mathbf{c}^{(0)}\big|_{\Pi^{(0)}} \leftarrow \mathbf{\Phi}^{\dagger}_{\Pi^{(0)}}\tilde{\mathbf{Y}}$
2: $\mathbf{r}^{(0)} \leftarrow \tilde{\mathbf{Y}} - \mathbf{\Phi}\mathbf{c}^{(0)}$
3: $T \leftarrow K^{(0)} + \Delta s;\quad k \leftarrow 1;\quad j \leftarrow 1$

Iterations:
4: **repeat**
5: $S_k \leftarrow \max(\mathbf{\Phi}^H \mathbf{r}^{(k-1)}, T - K^{(0)})$ {Preliminary test}
6: $C_k \leftarrow \Pi^{(k-1)} \cup S_k$ {Make candidate list}
7: $\Pi_t \leftarrow \max(\mathbf{\Phi}^{\dagger}_{C_k}\tilde{\mathbf{Y}}, T)$ {Temporary final list}
8: $\mathbf{c}^{(k)}\big|_{\Pi_t} \leftarrow \mathbf{\Phi}^{\dagger}_{\Pi_t}\tilde{\mathbf{Y}},\ \mathbf{c}^{(k)}\big|_{\Pi^c_t} \leftarrow \mathbf{0}$
9: $\mathbf{r} \leftarrow \tilde{\mathbf{Y}} - \mathbf{\Phi}_{\Pi_t}\mathbf{\Phi}^{\dagger}_{\Pi_t}\tilde{\mathbf{Y}}$ {Compute residue}
10: **if** $\|\mathbf{r}\|_2 \geq \left\|\mathbf{r}^{(k-1)}\right\|_2$ **then**
11: $j \leftarrow j+1, T \leftarrow K^{(0)} + j \times \Delta s$ {Stage switching}
12: **else**
13: $\Pi^{(k)} \leftarrow \Pi_t, \mathbf{r}^{(k)} \leftarrow \mathbf{r},$
14: $k \leftarrow k + 1$ {Same stage, next iteration}
15: **end if**
16: **until** $\|\mathbf{r}\|_2 < \boldsymbol{\varepsilon}$

Output:
Recovered clipping noise vector $\mathbf{c}$, s.t.
$\mathbf{c}|_{\Pi_i} = \mathbf{\Phi}^{\dagger}_{\Pi_i}\tilde{\mathbf{Y}},\ \mathbf{c}|_{\Pi^c_i} = \mathbf{0}$

2) Accuracy: First, as described in Line 1 of Algorithm 1, the PA-SAMP has a more accurate initial estimation of the sparse vector $\mathbf{c}^{(0)}\big|_{\Pi^{(0)}} \leftarrow \mathbf{\Phi}^{\dagger}_{\Pi^{(0)}}\tilde{\mathbf{Y}}$, which exploits the partial support instead of the trivial zero vector adopted in the SAMP. Second, as represented in Line 5 of Algorithm 1, in each iteration of the PA-SAMP, only $T - K^{(0)}$ new entries are necessarily identified in the preliminary test and merged with the previous temporary final list, while the $K^{(0)}$ initial entries acquired from the partial support are remained in the candidate list. This makes the iterations of the PA-SAMP more accurate. Moreover, during the stage switching, the PA-SAMP could adopt smaller step size Δs for its smaller gap to the actual sparsity level than the SAMP, which leads to more accurate estimation of the unknown sparsity level.

F. Final Clipping Noise Recovery and Symbols Estimation

From (4), it is notable that the clipping noise $\mathbf{c} \leq 0$. Therefore, $\mathbf{c_1}$ and $\mathbf{c_2}$ can be reconstructed by

$$\begin{cases} c_{1,n} = 0, c_{2,n} = \tilde{c}_n, & \text{if } \tilde{c}_n > 0, \\ c_{1,n} = \tilde{c}_n, c_{2,n} = 0, & \text{if } \tilde{c}_n \leq 0. \end{cases} \tag{16}$$

Then, the clipping noise is estimated as $\hat{\mathbf{C}} = [\mathbf{c_1}; \mathbf{c_2}]$.

After subtracting the estimated clipping noise $\hat{C}_k$ from the received symbol Y in the frequency domain, the final

TABLE I
AVERAGE NUMBER OF ITERATIONS

Sparsity Level	PA-SAMP	SAMP
4	1.74	3.36
6	3.24	5.28
8	4.02	7.14

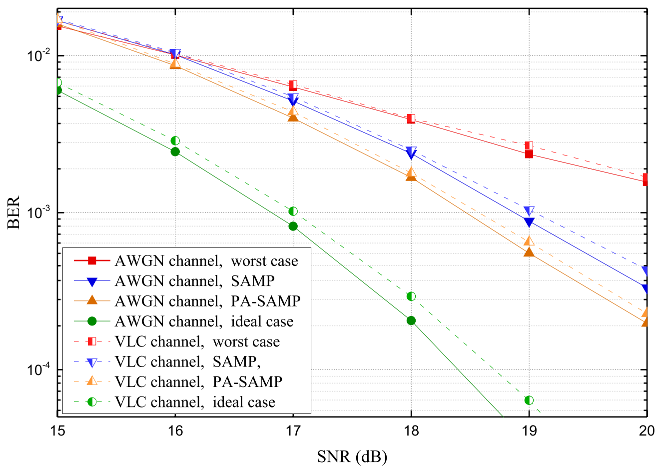

Fig. 2. The BER performance versus SNR when $N = 256$, 16-QAM and $A_{th} = \sqrt{2}$ are used.

estimation $\tilde{X}_k$ is obtained by

$$\tilde{X}_k = \arg\min |2(H_k{}^{-1}Y_k - \hat{C}_k) - s|, \ \ s \in \mathcal{X}, \ \ k \text{ is odd.} \quad (17)$$

IV. SIMULATION RESULTS

In this section, simulations are conducted to evaluate the performance of the proposed priori aided CS-based clipping noise cancellation scheme for ACO-OFDM systems with the bandwidth of 20 MHz.

In order to quantitatively measure the execution time of the PA-SAMP and SAMP, the average number of iterations using the PA-SAMP and SAMP for the clipping noise reconstruction is summarized in Table I, where the average value is calculated from 10^4 times of simulations. As can be seen from this table, the average iteration number of the PA-SAMP is much smaller than that of the SAMP in the case of different sparsity levels, which validates the theoretical analysis of complexity reduction in Section III.

The bit error rate (BER) of the ACO-OFDM systems is computed and compared with the conventional SAMP algorithm. The worst case ignoring the clipping noise and the ideal case without the clipping noise are also depicted as benchmarks. The BER performances under both the AWGN and VLC channels are simulated. The simulation parameters are configured with sub-carrier number $N = 256$, the clipping threshold $A_{th} = \sqrt{2}$, and 16-QAM and 64-QAM constellations for Fig. 2 and Fig. 3, respectively. At the target BER of 10^{-3}, the proposed scheme outperforms the conventional CS-based method by approximately 0.4 dB and 0.5 dB under the VLC channel in Fig. 2 and Fig. 3, respectively. Compared to the AWGN channel, the BER performances under the VLC channel are degraded about 0.15 dB and 0.2 dB at the BER of 10^{-3} for 16-QAM and

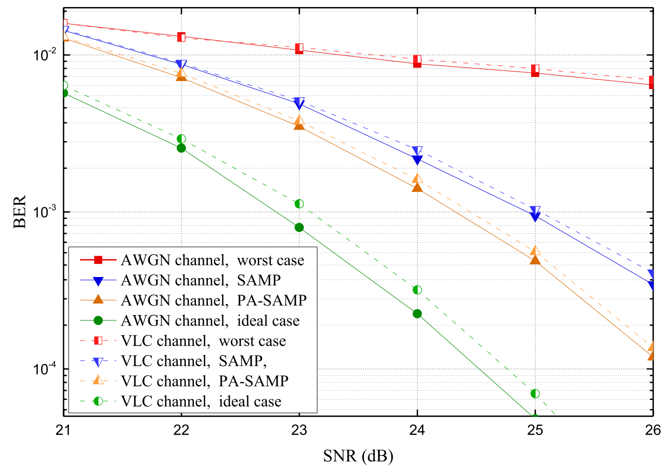

Fig. 3. The BER performance versus SNR when $N = 256$, 64-QAM and $A_{th} = \sqrt{2}$ are used.

64-QAM constellations in Fig. 2 and Fig. 3, respectively. Furthermore, the gaps between the proposed method and the ideal case are about 1.6 dB and 1.4 dB under the VLC channel in Fig. 2 and Fig. 3, respectively, which validates the accuracy and effectiveness of the proposed recovery method.

V. CONCLUSIONS

In this letter, the proposed method solves the RIP problem of the sensing matrix by modifying the CS equation, and effectively estimates the non-positive clipping noise for ACO-OFDM systems. Compared with the conventional schemes, with the support of the priori information, the proposed PA-SAMP algorithm is improved in both accuracy and robustness to reconstruct the clipping noise for ACO-OFDM systems with low computational complexity.

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