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Channel estimation for wideband underwater visible light communication: a compressive sensing perspective

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Abstract: With the rapid development of light emitting diode (LED), visible light communication (VLC) becomes an important technique for information transmission including underwater applications. However, accurate channel estimation for underwater VLC is still challenging due to the complex environment of the underwater VLC channel. In this paper, by utilizing a proper approximation, where the channel attenuation is linear with the frequency, a new compressive sensing (CS) based channel estimation approach is proposed. Utilizing the sparse property of the reflection path length for the underwater VLC channel, the CS framework is modeled to estimate the reflection path length, which can further recover the underwater VLC channel. Moreover, a Bayesian CS recovery algorithm is investigated to overcome the problem of high coherence for the sensing matrix which outperforms the conventional greedy algorithm such as orthogonal matching pursuit (OMP). Simulation results illustrate that our proposed channel estimation for underwater VLC systems has a superior performance which can significantly reduce the pilot overhead, improve the spectral efficiency, and enhance the estimation accuracy.

OCIS codes: (060.4510) Optical communications; (010.7340) Water; (290.0290) Scattering; (230.3670) Light-emitting diodes.

References and links

1. C. H. Yeh, Y. L. Liu, and C. W. Chow, "Real-time white-light phosphor-LED visible light communication (VLC) with compact size," Opt. Express **21**(22), 26192–26197 (2013).
2. "IEEE standard for local and metropolitan area networks-part 15.7:short-range wireless optical communication using visible light," (2011).
3. L. Zeng, D. O'brien, H. Minh, G. Faulkner, K. Lee, D. Jung, Y. Oh, and E. T. Won, "High data rate multiple input multiple output (MIMO) optical wireless communications using white LED lighting," IEEE J. Sel. Areas Commun. **27**(9), 1654–1662 (2009).
4. J. Grubor, O. Jamett, J. Walewski, S. Randel, and K.-D. Langer, "High-speed wireless indoor communication via visible light," ITG Fachbericht (2007).
5. H. Kaushal and G. Kaddoum, "Underwater optical wireless communication," IEEE Access **4**(1), 1518–1547 (2016).
6. H. M. Oubei, C. Li, K.-H. Park, T. K. Ng, M.-S. Alouini, and B. S. Ooi, "2.3 gbit/s underwater wireless optical communications using directly modulated 520 nm laser diode," Opt. Express **23**(16), 20743–20748 (2015).
7. H. M. Oubei, J. R. Duran, B. Janjua, H.-Y. Wang, C.-T. Tsai, Y.-C. Chi, T. K. Ng, H.-C. Kuo, J.-H. He, M.-S. Alouini, G.-R. Lin, and B. S. Ooi, "4.8 gbit/s 16-QAM-OFDM transmission based on compact 450-nm laser for underwater wireless optical communication," Opt. Express **23**(18), 23302–23309 (2015).
8. M. A. Khalighi, T. Hamza, S. Bourennane, P. LÂÍÂęon, and J. Opderbecke, "Underwater wireless optical communications using silicon photo-multipliers," IEEE Photon. J. **9**(4), 1–10 (2017).
9. W. Wang, P. Wang, T. Cao, H. Tian, Y. Zhang, and L. Guo, "Performance investigation of underwater wireless optical communication system using M-ary OAMSK modulation over oceanic turbulence," IEEE Photon. J. **9**(5), 1–15 (2017).
10. Y. Zhao, J. Xu, A. Wang, W. Lv, L. Zhu, S. Li, and J. Wang, "Demonstration of data-carrying orbital angular momentum-based underwater wireless optical multicasting link," Opt. Express **25**(23), 28743–28751 (2017).

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11. B. Cochenour, L. Mullen, and J. Muth, "Effect of scattering albedo on attenuation and polarization of light underwater," Opt. Lett. **35**(12), 2088–2090 (2010).
12. F. Akhoundi, J. A. Salehi, and A. Tashakori, "Cellular underwater wireless optical CDMA network: Performance analysis and implementation concepts," IEEE Trans. Commun. **63**(3), 882–891 (2015).
13. W. Hou, E. Jarosz, S. Woods, W. Goode, and A. Weidemann, "Impacts of underwater turbulence on acoustical and optical signals and their linkage." Opt. Express **21**(4), 4367–4375 (2013).
14. L. Lu, X. Ji, and Y. Baykal, "Wave structure function and spatial coherence radius of plane and spherical waves propagating through oceanic turbulence," Opt. Express **22**(22), 27112–27122 (2014).
15. Y. Zhang, X. Ji, X. Li, and H. Yu, "Thermal blooming effect of laser beams propagating through seawater," Opt. Express **25**(6), 5861–5875 (2017).
16. D. Donoho, "Compressed sensing," IEEE Trans. Inf. Theory **52**(4), 1289–1306 (2006).
17. J. Meng, W. Yin, Y. Li, N. T. Nguyen, and Z. Han, "Compressive sensing based high-resolution channel estimation for OFDM system," IEEE J. Sel. Topics Signal Process. **6**(1), 15–25 (2012).
18. Z. Han, H. Li, and W. Yin, *Compressive Sensing for Wireless Networks,* Cambridge University Press, UK, (2013).
19. M. F. Duarte and Y. C. Eldar, "Structured compressed sensing: From theory to applications," IEEE Trans. Signal Process. **59**(9), 4053–4085 (2011).
20. W. Ding, F. Yang, C. Pan, L. Dai, and J. Song, "Compressive sensing based channel estimation for OFDM systems under long delay channels," IEEE Trans. Broadcast. **60**(2), 313–321 (2014).
21. W. Ding, F. Yang, W. Dai, and J. Song, "Time-frequency joint sparse channel estimation for MIMO-OFDM systems," IEEE Commun. Lett. **19**(1), 58–61 (2015).
22. X. Ma, F. Yang, W. Ding, and J. Song, "Novel approach to design time-domain training sequence for accurate sparse channel estimation," IEEE Trans. Broadcast. **62**(3), 512–520 (2016).
23. X. Ma, F. Yang, S. Liu, J. Song, and Z. Han, "Design and optimization on training sequence for mmwave communications: A new approach for sparse channel estimation in massive MIMO," IEEE J. Sel. Areas Commun. **35**(7), 1486–1497 (2017).
24. Z. Wang and G. B. Giannakis, "Wireless multicarrier communications," IEEE Trans. Signal Process. **17**(3), 29–48 (2000).
25. A. Morel, B. Gentili, M. Chami, and J. Ras, "Bio-optical properties of high chlorophyll case 1 waters and of yellow-substance-dominated case 2 waters," Deep-Sea Research Part I **53**(9), 1439–1459 (2006).
26. M. Ahmed and L. Lampe, "Power line communications for low-voltage power grid tomography," IEEE Trans. Commun. **61**(12), 5163–5175 (2013).
27. S. Ji, Y. Xue, and L. Carin, "Bayesian compressive sensing," IEEE Trans. Signal Process. **56**(6), 2346–2356 (2008).
28. J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," IEEE Trans. Inf. Theory **53**(12), 4655–4666 (2007).
29. D. Needell and J. A. Tropp, "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples," Applied and Computational Harmonic Analysis **26**(3), 301–321 (2009).
30. E. J. Candes and T. Tao, "Decoding by linear programming," IEEE Trans. Inf. Theory **51**(12), 4203–4215 (2005).
31. Z. Ben-Haim, Y. C. Eldar, and M. Elad, "Coherence-based performance guarantees for estimating a sparse vector under random noise," IEEE Trans. Signal Process. **58**(10), 5030–5043 (2010).
32. D. Wipf and B. Rao, "Sparse Bayesian learning for basis selection," IEEE Trans. Signal Process. **52**(8), 2153–2164 (2004).

1. Introduction

With the rapid development of light emitting diode (LED), visible light communication (VLC) [1, 2], which can work in a wide bandwidth and high data rate [3], is attracting widespread concern by both academia and industry. Apart from the indoor application for VLC [4], the outdoor environments including terrestrial, space, and underwater links are also suitable scenarios for VLC applications. There are numbers of underwater activities including pollution monitoring, oil control, offshore explorations, and so on [5], whose realization needs high data rate communications. Although the acoustic communication underwater is highly concerned, it is limited by its narrow bandwidth and low data rate. Fortunately, VLC can provide a much higher data rate, and thus can be a promising solution in the future underwater communications [6–9].

Although the underwater VLC can provide a wide bandwidth, low latency, and robustness against multi-path propagation and Doppler spread [5, 10], the application for underwater VLC is still challenging. The attenuation of light transferred underwater is dependent on the wavelength [11, 12], where the attenuation of the signal increases with the frequency and is heavily attenuated by the sea water. Moreover, it is also affected by other propagation effects such as temperature fluctuations, salinity, scattering, dispersion, and beam steering [13–15].

These problem will cause a severe frequency selective property for the VLC channel. Therefore, the propagation channel of the underwater VLC applications is complicated, whose model and performance were investigated in [5]. As the channel estimation is a prerequisite to guarantee system performance, the complicated channel model will lead to the difficulty of channel estimation, which limits extensive utilization for underwater VLC especially for wideband systems.

Recently, as the study of compressive sensing (CS) is becoming a research hotspot [16–18], researchers can recover a high dimensional signal from a relatively small dimensional signal. In wireless communications, as the channel impulse response (CIR) can be modeled as a sparse vector [19–21], the channel estimation can be performed under the framework of compressive sensing (CS) [22, 23], which can achieve high estimation accuracy with high spectral efficiency. However, the underwater VLC channel is quite different with the wireless channel. The sparsity in time domain does not exist for the severe underwater VLC channel with wavelength-dependent attenuation. Therefore, a novel approach should be investigated to solve the underwater VLC channel estimation problem.

In this paper, a CS model is built for the underwater VLC channel estimation. Instead of time domain sparsity, we turn our concentration to the sparse nature of the reflection path length for VLC channels. As the number of underwater reflected paths is rather limited, their length can be considered sparse after the quantification. According to this CS model, a Bayesian CS recovery algorithm is utilized to recover the path length to overcome the high coherence problem for the sensing matrix. In this way, the underwater VLC channel can be recovered with high accuracy. Simulation results demonstrate that the proposed scheme has a superior performance in channel estimation and can reduce considerable amount of pilots. Moreover, the proposed Bayesian CS based scheme for underwater VLC channel estimation outperforms the conventional greedy algorithm such as orthogonal matching pursuit (OMP), which is an excellent solution for CS problem with high coherence sensing matrix.

The rest of the paper is organized as follows. The model of the underwater VLC channel and the corresponding system are investigated in Section II. The proposed channel estimation scheme utilizing the sparse character is presented in Section III. The Bayesian CS algorithm for underwater VLC channel estimation is introduced in Section IV. Simulation results and discussion are provided in Section V. Finally, the paper is concluded in Section VI.

Notation: Uppercase and lowercase boldface letters denote matrices and column vectors, respectively; $\lfloor\cdot\rfloor$ represents the operation of performing a decimal to a round down integer. $(\cdot)^T$, $\|\cdot\|_p$, and $\mathrm{E}(\cdot)$ denote transpose, the ℓ_p-norm of a vector, and the expectation of a random variable, respectively. $[\cdot]_i$ and $[\cdot]_{i,j}$ denote the i-th column/element of a matrix/vector, and (i, j)-th entry of a matrix, respectively.

2. System model and channel model

In this paper, the underwater VLC channel estimation is based on an orthogonal frequency division multiplexing (OFDM) system [24]. One OFDM frame in time domain is denoted by

$$\mathbf{t} = [\mathbf{c}^T, \mathbf{u}^T]^T, \tag{1}$$

where $\mathbf{c} = [c_0, c_0, \ldots, c_{M-1}]^T$ and $\mathbf{u} = [u_0, u_1, \ldots, u_{K-1}]^T$ are the M-length guard interval (GI) and K-length OFDM symbol, respectively. The GI is utilized to protect the symbol from being contaminated by the previous frame. The padding types of the GI fall into three categories, which are cyclic prefix (CP), zero padding (ZP), and pseudorandom noise (PN) sequence. In this paper, CP-OFDM is adopted for convenience to simplify the elimination of the inter block interference (IBI).

In frequency domain, the OFDM symbol is combined by multiple orthogonal subcarriers. Most of the subcarriers are used for transmitting data, while a few of them are pilot subcarriers

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and used for underwater VLC channel estimation. The reduction of pilot subcarriers amount can improve the spectral efficiency, but may also degrade the accuracy of channel estimation. Therefore, the tradeoff of spectral efficiency and estimation accuracy is essential and should be thoroughly considered.

The underwater VLC channel frequency response (CFR) is modeled as [5]

$$H(f) = \alpha e^{-\beta(f)s}, \tag{2}$$

where α is the attenuation factor, which is independent with frequency and can be set as 1 without loss of generality. The exponential term $\beta(f)$ denotes the cumulative attenuation coefficients, which is the effect for both the absorption and scattering. Parameter s is the distance travelled by the light signal. It can be seen that different frequencies of signal have different channel attenuations. Therefore, the underwater VLC channel is wavelength dependent, which will make the channel estimation more difficult.

The channel model above is based on only the line-of-sight (LOS) links. However, if there exist some shelters or echoes, the received signal may be the superposition of multiple paths. Under this condition, the channel model can be rewritten as

$$H(f) = \sum_{l=0}^{L-1} \alpha e^{-\beta(f)s_l}, \tag{3}$$

where s_l is the distance of the l-th path. The frequency f can be divided into two parts, i.e., $f = f_b + f_c$, where f_b and f_c are the baseband signal frequency and the modulation carrier frequency, respectively. L is the number of the paths. It should be mentioned that, there is not many shelters or echoes for underwater environment. Therefore, the parameter L is usually very small and this makes s_l sparse in the distance domain, which is the domain corresponding to the distance of the paths.

To recover the underwater VLC channel, the attenuation coefficient $\beta(f)$ is the key point, which is usually considered as not regular with the frequency. A typical attenuation trend in sea water for the visible light is shown in Fig. 1. It is investigated in Fig. 6(a) of literature [25], whose axis of wavelength is transformed to frequency in this paper. It can be seen that the attenuation coefficient can hardly be represented as a closed form equation for the whole spectrum. However, if we select only a small portion of the entire light-wave spectrum, which is still broadband compared to the existing radio frequency communication system, the relationship between the attenuation coefficient and the frequency can be approximately determined [11].

In our application, the utilized frequency bandwidth is only dozens of MHz, where the linear relationship is basically fitted [5]. Hence, we can model the attenuation coefficient as a linear model, which can be written as

$$\beta(f) = \beta_1 f + \beta_2, \tag{4}$$

where β_1 and β_2 are linear factors of the underwater VLC channel model, respectively. As a result, the underwater VLC channel is simplified and can be represented by

$$H(f) = \sum_{l=0}^{L-1} \alpha e^{-(\beta_1 f + \beta_2)s_l}. \tag{5}$$

Although the attenuation coefficient is still dependent with the frequency, the underwater VLC channel model can be characterized now by a closed form equation. In this way, the CS model can be formulated according to the sparsity in the distance domain, which is introduced in the next section.

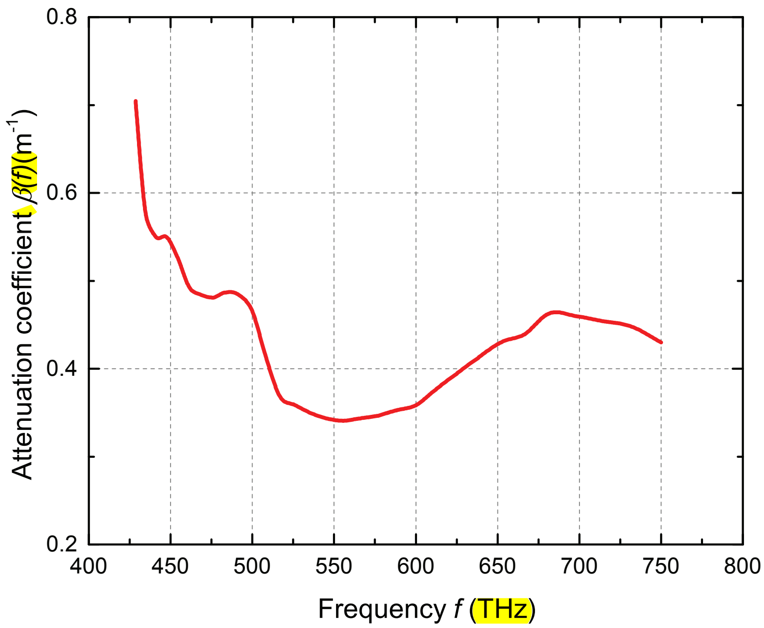

Fig. 1. A typical attenuation trend in sea water [25].

3. Proposed channel estimation

Assuming a discrete value for s_l, the reflection path length for the underwater VLC channel is sparse in the distance domain. The pilots for the OFDM frame is configured into uniformly-spaced P subcarriers in order to acquire equidistant CFR samplings ranging from $f_{\min}$ to $f_{\max}$. The sampling interval, which is also the interval of the pilots, is given by

$$\Delta f = \frac{f_{\max} - f_{\min}}{P-1}, \tag{6}$$

Based on this sampling method for the CFR, the i-th sampling frequency point is denoted as

$$f_i = f_{\min} + i\Delta f, \tag{7}$$

where $0 \leq i < P$. The corresponding CFR for the i-th sampling frequency point is represented by

$$\begin{aligned} H(f_i) &= \sum_{l=0}^{L-1} \alpha e^{-(\beta_1 f_i + \beta_2) s_l} \\ &= \sum_{l=0}^{L-1} \alpha e^{-[\beta_1 (f_{\min} + i\Delta f) + \beta_2] s_l} \\ &= \sum_{l=0}^{L-1} \lambda_l \cdot \omega_l^i, \end{aligned} \tag{8}$$

where λ_l and ω_l are written as

$$\lambda_l = \alpha e^{-(\beta_1 f_{\min} + \beta_2) s_l}, \tag{9}$$

and

$$\omega_l = e^{-\beta_1 \Delta f s_l}, \tag{10}$$

respectively. If the values of λ_l are all acquired for $0 \leq l < L$, s_l can be recovered by utilizing (9). By this means, the underwater VLC channel could be recovered which can be used at the receiver for the equalization. Under this circumstances, the channel estimation problem is converted into a parameter recovery issue with exponential accumulation. For traditional approaches, the subspace algorithm [26] is optional without considering the potential sparsity of the channel. To investigate the advantage of the sparse nature in distance domain, the CS framework is formulated in the following derivation.

The sampling frequency response can be achieved by allocating frequency pilots in the OFDM subcarriers. The uniformly-spaced CFR can be acquired by comparing the value of the transmitted and received pilots. The sampling frequency response can be regarded as a rough channel estimation. Once the rough estimation is achieved, the uniformly-spaced $H(f_i)$ for the sampling interval Δf can be combined as the measurement $\mathbf{y}$, which is given by

$$\mathbf{y} = [H(f_0), H(f_1), ..., H(f_{P-1})]^T. \tag{11}$$

The measurement $\mathbf{y}$ can be transformed to the product of a sensing matrix and a vector. If the vector is sparse and the sensing matrix is a flat matrix, whose height is much smaller than width, the CS structure can be built.

The key difficulty to build the CS structure is that the distance s_l is a continuous coefficient. It is known that CS theory is only applicable in discrete scenario. Therefore, the continuous distance has to be quantified firstly. In this condition, the minimum unit of the path Δs is important for distance discretization. The relationship between Δs and s_l is expressed as

$$s_l = n_l \Delta s, \tag{12}$$

where n_l is the quantization coefficient. The value of Δs should be carefully selected. If a small value of Δs is chosen, a high recovery accuracy can be achieved because the distance representation is more detailed. However, the computational complexity is increased and vice versa. Accordingly, a compromise plan need to be put forward considering both the accuracy and the computational complexity. After all, the CS model is presented as

$$\mathbf{y} = \mathbf{\Phi x}, \tag{13}$$

where the sparse vector $\mathbf{x}$ and the sensing matrix $\mathbf{\Phi}$ are written by,

$$\mathbf{x} = [x_0, x_1, \cdots, x_{N-1}]^T, \tag{14}$$

$$\mathbf{\Phi} = \begin{bmatrix} v_0 & v_1 & \cdots & v_{N-1} \\ v_0^2 & v_1^2 & \cdots & v_{N-1}^2 \\ \vdots & \vdots & \ddots & \vdots \\ v_0^P & v_1^P & \cdots & v_{N-1}^P \end{bmatrix}, \tag{15}$$

where x_k is equal to 0 or corresponds to λ_l. Coefficient N is determined by the maximum channel length $s_{\max}$ satisfying $N \geq s_{\max}/\Delta s$. v_k in (15) is the quantified variable from ω_l and can be denoted as

$$v_k = e^{-\beta_1 \Delta f \Delta s \cdot k}. \tag{16}$$

As $\mathbf{x}$ is a column vector with only $L (0 < L \ll N)$ nonzero entries, $\mathbf{x}$ has the property of sparse. For the nonzero entries, if the entry with index of m in $\mathbf{x}$ is nonzero, e.g., $c(m) \neq 0$, there exist a reflection paths with length $m\Delta s$. Therefore, the support $\mathcal{S}$, which is composed of the nonzero indices, is associated with the channel path, and is given by

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$$\mathcal{S} = \left\lfloor \left\{ \frac{s_l}{\Delta s} \right\}_{l=0}^{L-1} \right\rfloor. \tag{17}$$

Under these circumstances, s_l can be easily obtained by recovering the entries in $\mathcal{S}$. The channel path length is finally derived by

$$s_l = \mathcal{S}_l \cdot \Delta s, \tag{18}$$

where $\mathcal{S}_l$ is the l-th entry in $\mathcal{S}$.

4. Bayesian CS recovery algorithm

Equation (13) is a typical CS model. To find the solutions for the underdetermined equation, several different CS recovery algorithms are previously investigated, including the greedy algorithm and the Bayesian CS approach [27]. Orthogonal matching pursuit (OMP) [28] and compressive sampling matching pursuit (CoSaMP) [29] are representative greedy algorithms. In this framework, the so-called restricted isometry property (RIP) [30, 31] is one significant criterion to evaluate the performance of the sensing matrix. Nevertheless, the identification algorithm with polynomial time complexity for RIP does not exist [31]. Fortunately, the mutual incoherence property (MIP), which is the sufficient condition for RIP, can be easily evaluated instead. The optimized parameter, i.e., the *coherence* of the sensing matrix $\mathbf{\Phi} \in C^{P \times N}$ is the maximum coherence between different columns and is defined as

$$\mu = \max_{0 \le j,k \le N-1, j \ne k} \frac{\left|\left\langle \boldsymbol{\varphi}_j, \boldsymbol{\varphi}_k \right\rangle\right|}{\left\|\boldsymbol{\varphi}_j\right\|_2 \cdot \left\|\boldsymbol{\varphi}_k\right\|_2}, \tag{19}$$

where $\boldsymbol{\varphi}_j$ and $\boldsymbol{\varphi}_k$ are the j-th and the k-th column of the sensing matrix, respectively.

However, according to (15), it can be easily verified that the coherence of the sensing matrix $\mathbf{\Phi}$ is very large and approach to 1. Therefore, the traditional greedy CS recovery algorithms will probably be failed. In this condition, we choose Bayesian CS algorithm instead, which is also a well-established CS algorithm and on the basis of relevance vector machine (RVM) [32]. Both the sparse vector and the noise are characterized based on the set up of a statistical model. When conducting the operation of matrix inversion, which is inevitable for CS recovery, it takes advantage of the auxiliary information of the noise to solve the problem for the large coherence value of the sensing matrix. The Bayesian approach does not include the recovery of the singular value of the sensing matrix, which is usually converted into a high order equation solving problem. Therefore, it tends to have a superior performance in the scenario of strong noise. Furthermore, the Bayesian approach requires fewer pilots than traditional ones due to the utilization of the sparse character of the underwater VLC channel. In summary, the algorithm has excellent anti-noise performance, which is beneficial to the increase of the spectral efficiency for the underwater VLC system.

The proposed Bayesian CS approach for underwater VLC channel estimation in this paper is illustrated in Algorithm **1**. The power of the signal is assumed to be a priori vector $\boldsymbol{\xi}^{(0)}$ as the initialization. The noise in the channel is described by a vector $\boldsymbol{w}^{(0)}$ to consider the influence of the noise, which is initialized to be proportional to the variance of the measurement $\mathbf{y}$. p is the threshold, which is utilized to obtain the zero support for each iteration. In order to guarantee the performance of the algorithm, considerable number of iterations is required. As the process goes on, the result will eventually converge. At that time, the algorithm ends and the recovery result of the sparse signal is obtained.

After we achieve vector $\mathbf{x}$, the λ_l, ω_l, and the corresponding α and s_l can be calculated. As a result, the underwater VLC channel is got according to (8).

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Algorithm 1 Bayesian CS for underwater VLC channel estimation.

Inputs:

1) Received pilots $\mathbf{y} \in C^{P}$;

2) Sensing matrix $\mathbf{\Phi} \in C^{P \times N}$;

Initialization:

The noninformative priori vector $\boldsymbol{\xi}^{(0)} = [1, 1, ..., 1] \in C^{N}$;

The noise modeling $\boldsymbol{w}^{(0)} = 0.01 \times \mathrm{var}(\mathbf{y})$;

The threshold p;

Maximum iteration time $t_{\max}$;

Iterations:

for $i = 0 : t_{\max} - 1$ **do**

  Select the column vectors $\boldsymbol{\phi}_j$ and the corresponding elements in $\boldsymbol{\xi}^{(i)}$, satisfying $\boldsymbol{\xi}_j^{(i)} < p$

  $\mathbf{\Psi}^{(i)} = \sum_j \boldsymbol{\xi}_j^{(i)} \boldsymbol{\phi}_j \boldsymbol{\phi}_j^{H} + \boldsymbol{w}^{(i)} \boldsymbol{I}$, where $\boldsymbol{I}$ is a unit matrix

  for $k = 0 : N - 1$ **do**

    $\boldsymbol{\rho}_k^{(i)} = \boldsymbol{\xi}_k^{(i)} \left(\mathbf{\Phi} \left(\mathbf{\Psi}^{(i)} \right)^{-1} \mathbf{y} \right)_k$

    $\boldsymbol{\varepsilon}_k^{(i)} = \boldsymbol{\xi}_k^{(i)} - \left(\boldsymbol{\xi}_k^{(i)} \right)^2 (\boldsymbol{\phi}_k)^H \left(\mathbf{\Psi}^{(i)} \right)^{-1} \boldsymbol{\phi}_k$

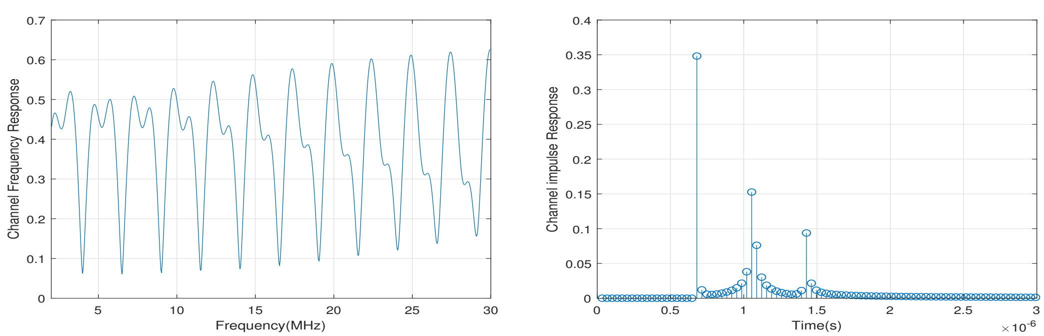
    $\boldsymbol{\xi}_k^{(i+1)} = \left\| \boldsymbol{\rho}_k^{(i)} \right\|_2^2 + \boldsymbol{\varepsilon}_k^{(i)}$

  end for

  $\boldsymbol{w}^{(i+1)} = \left(\left\| \mathbf{y} - \mathbf{\Phi} \boldsymbol{\rho}^{(i)} \right\|_2^2 + \sum_j \boldsymbol{\xi}_j^{(i+1)} \right) / P$

end for

Output: The sparse signal $\hat{\mathbf{x}} = \boldsymbol{\rho}^{(t_{\max}-1)}$



(a) (b)

Fig. 2. The snapshot of the typical 3-path underwater VLC equivalent baseband channel in frequency domain (a) and time domain (b).

5. Simulation results

The simulation results of the the proposed channel estimation for the underwater VLC channel is provided in the following. The parameters in the simulations are listed in Table 1.

Firstly, the equivalent baseband CFR and CIR for the underwater VLC channel are illustrated in Fig. 2. It is clear in Fig. 2 that, although the number of paths is limited, the CIR is not sparse in time domain. This implies that the traditional time-domain-sparsity based CS algorithm is no longer available. Moreover, according to Fig. 2, the underwater VLC channel is frequency

Table 1. Coefficients in the Simulations

Length of the GI M	256
Length of the OFDM symbol K	2048
Equivalent baseband central frequency	16 MHz
Equivalent baseband bandwidth $f_{\max} - f_{\min}$	28 MHz
Light carrier wavelength/frequency	450 nm/666.7 THz
Number of paths for VLC channel reflection L (including the line-of-sight link)	3

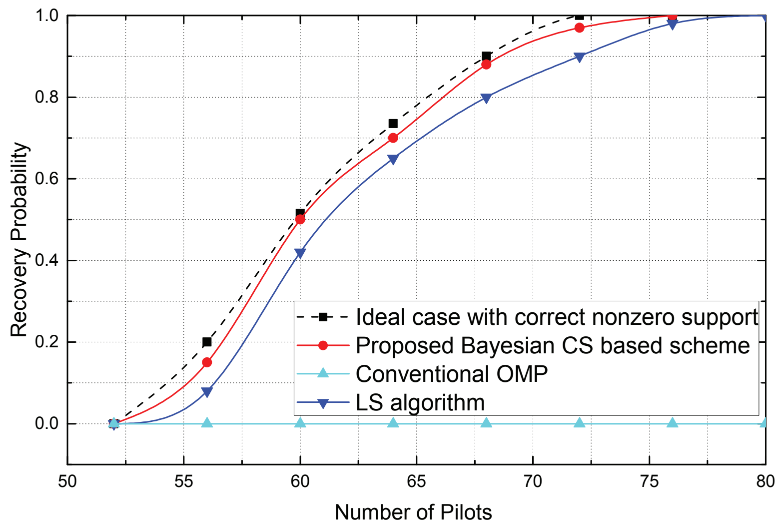


Fig. 3. Simulation for accurate recovery probabilities when SNR = 20 dB.

selective.

Figure 3 presents the accurate recovery probability of the CIR for the proposed channel estimation scheme. The performance of the least square (LS) algorithm is also shown as reference, which can be formulated as

$$\widehat{H}(f_i) = Y_i / X_i, \tag{20}$$

where Y_i and X_i are the received and transmitted pilot symbol on the i-th $(0 \leq i < P)$ pilot subcarriers, respectively. The traditional OMP algorithm, which is a widely used greedy CS recovery algorithm, fails to recover the sparse underwater VLC channel because the sensing matrix has large coherence. The accurate recovery here is set as the condition when the MSE is less than 10^{-2}. The signal-to-noise ratio (SNR) is configured to be 20 dB. Different numbers of pilots are simulated. It is obvious that the proposed Bayesian based CS scheme outperforms the LS algorithm, and it gets close to the ideal case, which assumes the correct nonzero support is known at the receiver. Our scheme achieves the accurate recovery probability of 0.9 at the pilot number of less than 68 when the SNR is 20 dB. The LS algorithm needs around 5 extra pilots. This shows that our scheme is more capable for underwater VLC channel estimation.

The mean square error (MSE) of CIR simulations for different schemes are illustrated in Fig. 4. On the one hand, the traditional OMP and LS algorithms are simulated by contrast. On the other hand, the Cramer-Rao lower bound (CRLB) is also evaluated as a reference. In the ideal situation, the non-zero support is completely recovered and is represented as $\mathbf{D}\,(\|\mathbf{D}\|_0 = L)$. Therefore, the

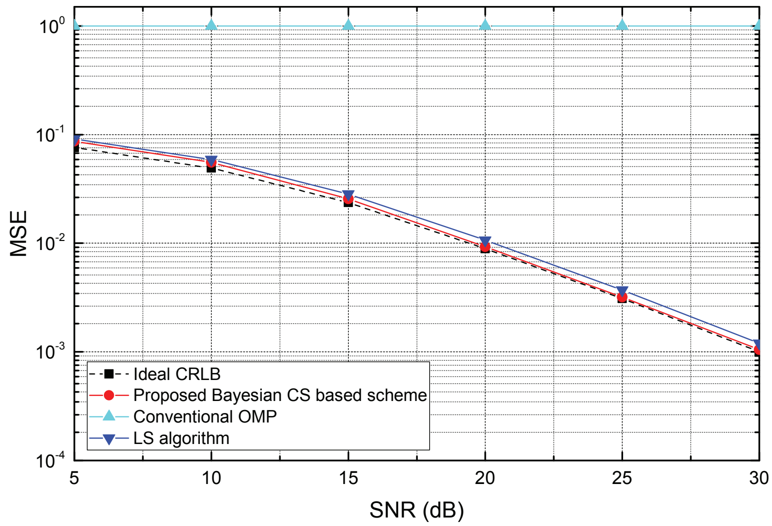


Fig. 4. The MSE performance of CIR simulations.

elements beyond the support $\mathbf{D}$ are configured as 0. In this condition, equation (13) is rewritten as

$$\mathbf{y} = \mathbf{\Phi}_D \mathbf{x}_D. \tag{21}$$

Utilizing the maximum likelihood criterion, the solution is given by

$$\mathbf{x}_{\text{est}} = \mathbf{\Phi}_D{}^{+}\mathbf{y} = \left(\mathbf{\Phi}_D{}^{H}\mathbf{\Phi}_D\right)^{-1}\mathbf{\Phi}_D\mathbf{y}. \tag{22}$$

Therefore, the CRLB of $\mathbf{x}$ is written as

$$\text{CRLB} = \text{E}\left\{\|\mathbf{x}_{\text{est}} - \mathbf{x}\|_2\right\} = L\sigma^2/P. \tag{23}$$

For other coefficients, the number of the pilots is configured as $P = 64$, while the minimum unit of the path is $\Delta s = 1m$. It is shown that our scheme has better performance than the OMP scheme, since it fails to recover the sparse underwater VLC channel due to the large coherence of the sensing matrix. The MSE gets close to the CRLB and is less than 0.1 dB away from it. Besides, around 1 dB improvement is obtained compared with the LS algorithm with 20 dB SNR. Moreover, in the high SNR level like 25 dB, the performance is almost the same with that of the ideal case. Therefore, the proposed Bayesian CS based scheme is efficient to solve the underwater VLC channel estimation problem.

6. Conclusions

In this paper, a new underwater VLC channel estimation method based on Bayesian CS is put forward. The channel estimation can be simplified to a coefficient estimation problem with exponential accumulation by adopting a linear approximation for the underwater VLC channel. Taking advantage of the sparsity in the distance domain, the CS model is formulated. To overcome the difficulty that the coherence of the sensing matrix is large, the Bayesian CS algorithm based on RVM is put forward. Simulations demonstrate that the our scheme outperforms the traditional OMP and LS methods in terms of the recovery probability and MSE of the CIR. Consequently, our scheme is a potential technique for underwater VLC channel estimation.

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