Spectrally Efficient CSI Acquisition for Power Line Communications: A Bayesian Compressive Sensing Perspective

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***Abstract*— Power line communication (PLC) techniques present a no extra wire solution for the communication purpose in a smart grid due to the ubiquity and low cost. Moreover, the through-the-grid property of PLC has naturally extended its possible applications, including but not limited to the automatic meter reading, line quality monitoring, online diagnostics, and network tomography. To guarantee the performance of communications as well as other applications in PLC systems, accurate channel state information (CSI) acquisition should be performed regularly. However, the conventional pilot-based CSI acquisition approaches in PLC systems have not made full use of the channel characteristics and hence suffer from a low spectral efficiency. In this paper, by exploiting the parametric sparsity and discretizing the electrical length in the well-known PLC channel model, we formulate the non-sparse (either time domain or frequency domain) PLC channel into a compressive sensing (CS) applicable problem. Furthermore, we propose a spectrally efficient CSI acquisition scheme under the framework of Bayesian CS and extend it to the multiple-input multiple-output PLC by investigating the channel spatial correlation. Compared with the existing sparse CSI acquisition schemes for PLC, such as the annihilating filter-based and the estimating signal parameters via rotational invariance technique-based ones, the proposed scheme has better mean square error performance and noise robustness.**

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I. INTRODUCTION

DUE TO the ubiquitous availability, easy installation and low cost, the broadband power line communication (PLC) has been considered as an appealing communication solution for both smart grid and smart home applications [1]–[4]. To accelerate the developments of broadband PLC, several competing standards have been proposed recently, such as Homeplug AV2.0 [5], G.hn [6], and IEEE 1901 [7]. By adopting the orthogonal frequency division multiplexing (OFDM), advanced coded modulation techniques, multiple input multiple output (MIMO) scheme, and effective noise mitigation approaches, broadband PLC can achieve a data rate of Gbps [5], [8]–[11].

In fact, PLC could be more than a means of communications. Thanks to its "through the grid" property, PLC can provide many other value-added services together with the communications [2], e.g., the automatic meter reading [12], line quality monitoring [13], online diagnostics [14], and network tomography [15]. In practice, accurate channel state information (CSI) acquisition is a prerequisite to guarantee the performance of communications and other services [13], [15], [16]. However, the PLC channel is time-varying due to the time-variant grid topology caused by the changes in the electrical loads. Consequently, the CSI acquisition of the PLC needs to be performed regularly.

Among the numerous CSI acquisition approaches for PLC systems [17]–[21], the training-based ones are the most mature and widely adopted. Generally, PLC systems use the pilots for CSI acquisition under the least squared (LS) criterion [18]–[21]. But the drawback is that it doesn't make use of channel characteristics to reduce the pilot overhead, and is not spectrally efficient. Some researchers [22] have directly applied the compressive sensing (CS) into the time-domain PLC channel model. Nevertheless, the PLC channel is not sparse in either time or frequency domain according to [3] and [23], which indicates that the straightforward application of CS would fail. Recently, by exploiting the parametric sparsity of PLC channels, the sparse CSI acquisition schemes based on the annihilating filter [24] and the estimating signal parameters via rotational invariance technique (ESPRIT) [15], have been proposed. Such schemes could significantly reduce the pilot overhead and achieve higher spectral efficiency, which opens a brand new field of vision for CSI acquisition in PLC systems. However, we found that due to the ill-conditioned root-finding and singular value decomposition (SVD) operations, these two schemes [15], [24] are sensitive to noise.

In this paper, we propose a noise-robust and spectral efficient sparse CSI acquisition scheme based on the Bayesian CS (BCS) [25], [26]. Specifically, the contributions of this paper can be summarized as follows:

- Firstly, we have well reviewed two representative sparse CSI acquisition schemes, i.e., the annihilating filter based [24] and ESPRIT based [15] ones, and discuss their noise sensitivity.
- Secondly, the noise-robust CS cannot be directly applied because the PLC channels demonstrate neither time-domain nor frequency-domain sparsity. By discretizing the electrical length with an appropriate resolution, we exploit the parametric sparsity of the PLC channel and formulate the CSI acquisition into a CS applicable problem.
- Thirdly, by evaluating the proposed observation matrix, we find that the conventional greedy CS cannot recover the desired CSI due to the strong coherence of the matrix. In this context, we propose a noise-robust and spectrally efficient sparse CSI acquisition scheme under the framework of BCS.
- Fourthly, by exploiting the spatial correlation, we propose two schemes for MIMO PLC CSI acquisition, based on the Multiple response model Sparse Bayesian Learning (MSBL) [27] and block SBL (BSBL) [28] algorithms. Specifically, a modified BSBL algorithm has been proposed to better utilize the correlation and cope with the channels of only partial common path delays.
- Finally, numerical simulations have been carried out to compare the performances of the different approaches. The proposed scheme demonstrates better performance as well as higher spectral efficiency than the existing schemes in both single input single output (SISO) and MIMO PLC systems.

The remainder of this paper is organized as follows. In Section II, we describe the abstract model of the SISO and MIMO PLC channels used throughout the paper and introduce the channel parametric sparsity as well as the spatial correlation property of MIMO PLC channels. The two representative sparse CSI acquisition schemes are reviewed and discussed in Section III. The proposed BCS based CSI acquisition schemes for SISO and MIMO PLC systems are described in Section IV and Section V, respectively. Section VI and Section VII address the discussions and simulation results. Finally, the conclusions are provided in Section VIII.

Notation: The uppercase and lowercase boldface letters are used to denote matrices and column vectors; $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, $(\cdot)^\dagger$, $diag(\cdot)$, $\|\cdot\|_{\mathcal{F}}$, $\|\cdot\|_p$, $\otimes$, $\mathrm{Tr}(\cdot)$, and $\lfloor\cdot\rfloor$ denote the transpose, conjugate transpose, matrix inversion, Moore-Penrose matrix inversion, diagonal matrix, Frobenius norm, l_p norm, Kronecker product, matrix trace, and floor operations, respectively; $\mathbf{x}|_{\mathcal{D}}$ denotes the entries of the vector $\mathbf{x}$ in the set of $\mathcal{D}$; $\mathbf{\Phi}|_{\mathcal{D}}$ represents the sub-matrix comprising the $\mathcal{D}$ columns of $\mathbf{\Phi}$.

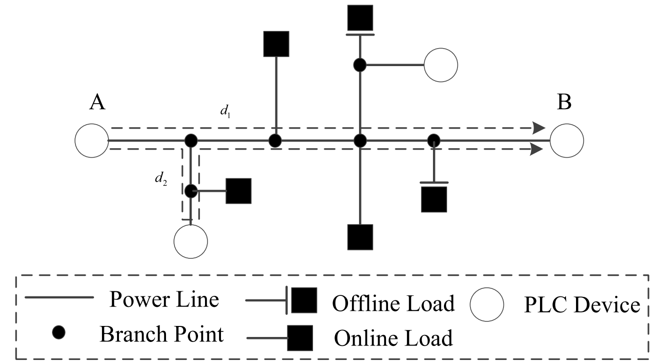

Fig. 1. The typical topology of a power line network.

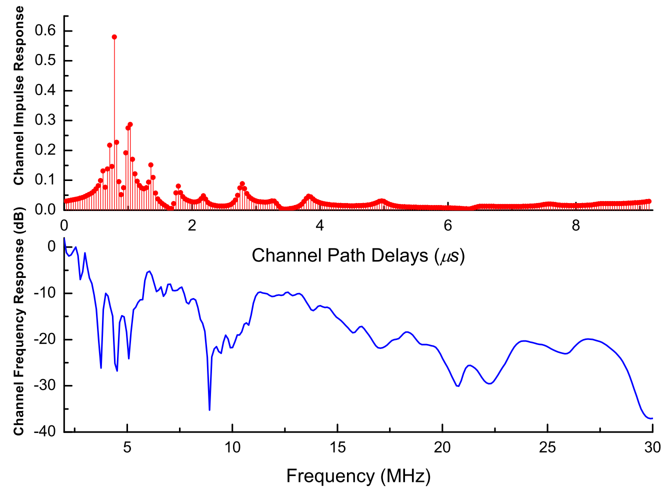

Fig. 2. The channel impulse response and channel frequency response of a typical 15-path PLC channel [23].

II. System Model

A. SISO PLC Channel

Among the classical PLC models [23], [29]–[31], the Zimmermann's model [23], which adopts a bottom-up modeling processing based on the transmission line theory, is the most widely used. The sparse CSI acquisition schemes for PLC, including the existing ones and our proposed one, are based on this model.

Due to the impedance mismatch in the power line network illustrated in Fig. 1, the PLC channel usually undergoes the "multipath" fading, whose channel frequency response can be represented as

$$H(f) = \sum_{l=1}^{L} g_l \cdot e^{-(\alpha_0+\alpha_1 f)d_l} \cdot e^{-j2\pi f \frac{d_l}{v_g}}, \tag{1}$$

where L is the number of paths, g_l and d_l represent the weighting factor and electrical length of the l-th path, α_0 and α_1 denote the attenuation parameters, while v_g is the group velocity of the power cable. Since the path of a longer distance usually suffers severer attenuation, the number of the paths L can be modeled to be finite and small [15], [23].

Parametric sparsity: Following the example of wireless communications, some researchers [22] have directly applied the CS into the time-domain PLC channel model. Nevertheless, according to (1), the PLC channel is not sparse in either time or frequency domain [3], [23], as depicted in Fig. 2,

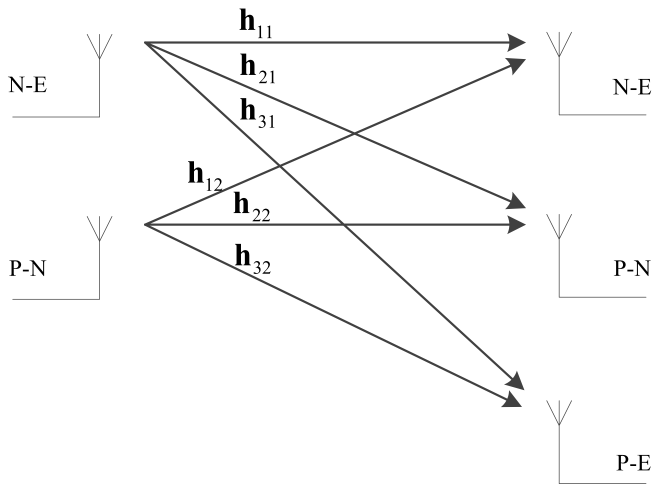

Fig. 3. A 2×3 PLC MIMO channel model.

which indicates that the straightforward application of CS would fail. Fortunately, it can be observed from (1) that for a deterministic channel, the number of parameters to be estimated[1] is far less than the channel dimension, i.e., the PLC channel has the so-called "***parametric sparsity***".

To exploit the parametric sparsity, we will have uniform samplings of the continuous channel frequency response between $[f_{\min}, f_{\max}]$ with a spacing Δf, and then the discrete channel frequency response can be written by

$$\begin{aligned} H_k &\triangleq H(f_k) = H(f_{\min} + k\Delta f) \\ &= \sum_{l=1}^{L} g_l \cdot e^{-[\alpha_0+\alpha_1(f_{\min}+k\Delta f)]d_l} \cdot e^{-j2\pi(f_{\min}+k\Delta f)\frac{d_l}{v_g}} + w_k \\ &= \sum_{l=1}^{L} c_l \cdot u_l^k + w_k, \end{aligned} \tag{2}$$

where c_l and u_l are the intermediate variables given by,

$$c_l = g_l \cdot e^{-[(\alpha_0+\alpha_1 f_{\min})d_l + j2\pi f_{\min}\frac{d_l}{v_g}]}, \tag{3}$$

$$u_l = e^{-(\alpha_1 d_l + j2\pi\frac{d_l}{v_g})\Delta f}, \tag{4}$$

and w_k represents the interference effect caused by the noise.

B. MIMO PLC Channels and Spatial Correlation Property

During the last few years, MIMO PLC techniques have been investigated to achieve higher data rates by utilizing the three-wire power cable for communications: Phase (P), Neutral (N) and Earth (E) [11]. The SISO PLC systems use only the P-N port to transmit and receive the signals. If the unused E wire is utilized, the three wires can form three ports P-N, P-E and N-E to constitute a 2×3 MIMO configuration[2] as shown in Fig. 3.

[1]The changes of power line network topology will only influence the L, g_l, and d_l. For simplicity, the other parameters could be considered as constants and obtained *in priori* under a deterministic power line network.

[2]Usually, due to Kirchhoff's law, only two input ports can be used simultaneously for a three-wire power cable. For all practical purposes, one can deploy any 2×2, 2×3 or 2×4 (with an additional common mode (CM, receive only) port [32]) MIMO configurations for the typical power cables.

Generally, for an $N_t \times N_r$ MIMO PLC systems with N_t transmit and N_r receive antennas, the transfer function between the port p of the transmit node and the port q of the receive node can be represented by [11], [33], [34]

$$H^{(pq)}(f) = \sum_{l=1}^{L} g_l^{(pq)} \cdot e^{-j\varphi^{(pq)}} \cdot e^{-(\alpha_0+\alpha_1 f)d_l} \cdot e^{-j2\pi f\frac{d_l}{v_g}}. \tag{5}$$

The model in (5) is mainly based on the assumption that the channels between different port pairs demonstrate the spatial correlation property, where only two parameters are different among the MIMO links: the path gains g_l and the correlation term $\varphi^{(pq)}$. This is because the topology of the power line network is the same, which is independent from the port pairs, and the signals between different port pairs will undergo the similar propagation paths [11]. This model has been derived based on the multi-conductor transmission line theory and validated via the field measurements [33], [34]. However, the key assumption of this model is that different channels share the common path delays, which will not hold in two cases: A) if the CM receive port is employed, since CM reception uses a unique signal transmission mechanism [11]; B) if not all conductors are tapped for some loads and only partial common delays exist. For Case A, it only needs to perform an individual CSI acquisition at the CM reception and is not worth studying. For Case B, it will introduce challenges to the current schemes and be of great research interests.

By exploiting the commonly used orthogonal pilots in the transmitted OFDM data block for different transmit antennas, i.e., allocating different start sampling frequency $f_{\min}^{(pq)}$ while using the same sampling spacing Δf, we could have the uniform samplings of the channel frequency responses, which are similar to the SISO case and given by

$$\begin{aligned} H_k^{(pq)} &\triangleq H(f_k^{(pq)}) = H(f_{\min}^{(pq)} + k\Delta f) \\ &= \sum_{l=1}^{L} g_l^{(pq)} \cdot e^{-j\varphi^{(pq)}} \cdot e^{-[\alpha_0+\alpha_1(f_{\min}^{(pq)}+k\Delta f)]d_l} \\ &\quad \cdot e^{-j2\pi(f_{\min}^{(pq)}+k\Delta f)\frac{d_l}{v_g}} + w_k^{(pq)} \\ &= \sum_{l=1}^{L} c_l^{(pq)} \cdot u_l^k + w_k^{(pq)}, \end{aligned} \tag{6}$$

where $c_l^{(pq)}$ is the intermediate variable given by,

$$c_l^{(pq)} = g_l^{(pq)} \cdot e^{-[(\alpha_0+\alpha_1 f_{\min}^{(pq)})d_l + j(2\pi f_{\min}^{(pq)}\frac{d_l}{v_g}+\varphi^{(pq)})]}, \tag{7}$$

and $w_k^{(pq)}$ represents the interference effect caused by the noise.

Remarks: In this way, the CSI acquisition of PLC systems can be transformed to the classical spectral estimation problem formulated in (2) and (6), and then solved under the framework of the finite rate of innovation (FRI) [15], [24]. In the next section, we will briefly review two representative FRI based sparse CSI acquisition schemes, i.e., the annihilating filter based [24] and the ESPRIT based [15] ones.

III. Sparse CSI Acquisition Schemes for PLC Systems: State-of-the-Art

In this section, we review the annihilating filter based and the ESPRIT based schemes for sparse CSI acquisition in PLC systems, and discuss why such schemes are sensitive to noise. Here we only discuss the methods in SISO cases for simplicity.

A. Annihilating Filter Based Scheme

For the signal $H_k = \sum_{l=1}^{L} c_l \cdot u_l^k$, its annihilating filter can be constructed as

$$\hat{A}(z) = \prod_{l=1}^{L} (1 - u_l z^{-1}) = \sum_{m=0}^{L} A_m z^{-m}, \tag{8}$$

where $A_m, m = 0, 1, \ldots, L$ are the filter coefficients.

According to the definition of the annihilating filter, the convolution between the signal H_k and the filter A_m will be zero, which is given by

$$\begin{aligned} \sum_{m=0}^{L} A_m H_{k-m} &= \sum_{m=0}^{L} A_m \sum_{l=1}^{L} c_l u_l^{k-m} \\ &= \sum_{l=1}^{L} c_l u_l^k \sum_{m=0}^{L} A_m u_l^{-m} = 0. \end{aligned} \tag{9}$$

Based on (9), the following equations can be constructed

$$\begin{cases} A_0 H_L + A_1 H_{L-1} + \cdots + A_L H_0 = 0 \\ A_0 H_{L+1} + A_1 H_L + \cdots + A_L H_1 = 0 \\ \qquad \vdots \\ A_0 H_{M-1} + A_1 H_{M-2} + \cdots + A_L H_{M-L-1} = 0. \end{cases} \tag{10}$$

Without loss of generality we can assume $A_0 = 1$, so that the matrix form of (10) considering the noise effect can be re-written as

$$\mathbf{Ha} = \mathbf{b} + \mathbf{w}, \tag{11}$$

where

$$\mathbf{H} = \begin{bmatrix} H_0 & H_1 & \cdots & H_{L-1} \\ H_1 & H_2 & \cdots & H_L \\ \vdots & \vdots & \ddots & \vdots \\ H_{M-L-1} & H_{M-L} & \cdots & H_{M-1} \end{bmatrix}, \tag{12}$$

$$\mathbf{a} = [A_L, A_{L-1}, \cdots, A_1]^T, \tag{13}$$

$$\mathbf{b} = -[H_L, H_{L+1}, \cdots, H_{M-1}]^T. \tag{14}$$

After the filter coefficients have been obtained, u_l can be derived by finding the filter roots,

$$\{u_l\}_{l=1}^{L} = \text{roots}\{[A_0, A_1, \cdots, A_L]\}, \tag{15}$$

and then the coefficients c_l can be obtained under the LS criterion. For a certain power line network, the parameters including α_0, α_1 and v_g are constant and known *a priori*, and hence the electrical length and the weighting factor can be calculated,

$$d_l = -\ln(u_l) v_g / [(\alpha_1 v_g + j2\pi)\Delta f], \tag{16}$$

$$g_l = c_l \cdot e^{(\alpha_0 + \alpha_1 f_{\min}) d_l + j2\pi f_{\min} \frac{d_l}{v_g}}. \tag{17}$$

B. ESPRIT Based Scheme

Consider again the matrix $\mathbf{H}$ in (12), and it can be decomposed as $\mathbf{H} = \mathbf{USV}^H$, where $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{V}^H$ are given by:

$$\mathbf{U} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ u_1 & u_2 & \cdots & u_L \\ \vdots & \vdots & \ddots & \vdots \\ u_1^{M-L-1} & u_2^{M-L-1} & \cdots & u_L^{M-L-1} \end{bmatrix}, \tag{18}$$

$$\mathbf{S} = diag(c_1, c_2, \cdots, c_L), \tag{19}$$

$$\mathbf{V}^H = \begin{bmatrix} 1 & u_1 & \cdots & u_1^{L-1} \\ 1 & u_2 & \cdots & u_2^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & u_L & \cdots & u_L^{L-1} \end{bmatrix}. \tag{20}$$

In practice, the matrix $\mathbf{H}$ will be decomposed using the unique SVD as

$$\mathbf{H} = \mathbf{U}_s \mathbf{S}_s \mathbf{V}_s^H + \mathbf{U}_n \mathbf{S}_n \mathbf{V}_n^H, \tag{21}$$

where the columns of $\mathbf{U}_s$ and $\mathbf{V}_s$ are L principal left and right singular vectors of $\mathbf{H}$, representing the signal subspace, while the second terms $\mathbf{U}_n \mathbf{S}_n \mathbf{V}_n^H$ represent the noise subspace.

It can be proved that both $\mathbf{U}_s$ and $\mathbf{V}_s$ will satisfy the shift-invariant property, and therefore, the estimates $\hat{u}_l$ for u_l is the eigenvalues of the matrix $\mathbf{Z} = \underline{\mathbf{U}_s}^{\dagger} \cdot \overline{\mathbf{U}_s}$ [15], [35], where $\underline{(\cdot)}$ and $\overline{(\cdot)}$ denote the operations of omitting the last and first row of $(\cdot)$, respectively.

After obtaining the u_l, the coefficients c_l can then be derived in the same way as the annihilating filter based scheme. Then the d_l and g_l can be calculated according to (16) and (17).

C. Noise Sensitivity

The annihilating filter based scheme could perfectly recovery the parameters from only $2L$ measurements in the noiseless scenario. However, in the noisy cases, this scheme will degrade rapidly due to the ill-conditioned root-finding operation in (15) even if with oversampling [35], [36].

The kernel operation of the ESPRIT algorithm is the SVD. Based on the principle of SVD, the L largest singular values and their singular vectors $\mathbf{U}_s$ correspond to the signal subspace, while the other singular values and vectors correspond to the noise subspace. If we could find the signal subspace correctly, the problem will be perfectly solved. However, if the singular values of the noise subspace is comparable to those of the signal subspace, for example, when the signal to noise ratio (SNR) is not high enough, the performance of the ESPRIT algorithm will degrade significantly [35], [36].

In the following sections, we will provide the simulation performance of the annihilating filter based and ESPRIT based schemes and show that they are sensitive to noise and might be impractical for CSI acquisition.[3]

[3]Moreover, the parameter L, which has to be exactly known *a priori* in annihilating filter and ESPRIT algorithms, is however unknown in the practice, limiting the feasibility of such methods.

IV. Bayesian Compressive Sensing Based Sparse CSI Acquisition

The CS theory has been proved to be robust to noise when recovering the sparse signals. However, we have shown that it is invalid to directly apply CS into the PLC CSI acquisition since the PLC channel is not sparse in either time or frequency domain. In order to apply the CS algorithms, we must first construct an observation matrix. In this paper, by discretizing the electrical length with a resolution Δd, i.e., $n_l = \lfloor d_l/\Delta d \rfloor$ and $N = \lfloor \max(d_l)/\Delta d \rfloor$, the continuous spectral estimation problem in (2) and (6) can be turned into a CS applicable discrete estimation problem.

Let us consider the SISO case again, and with the above discretization, the model in (2) can be represented by

$$\begin{aligned} H_k &= \sum_{l=1}^{L} g_l \cdot e^{-[\alpha_0+\alpha_1(f_{\min}+k\Delta f)]n_l\Delta d} \\ &\quad \cdot e^{-j2\pi(f_{\min}+k\Delta f)\frac{n_l\Delta d}{v_g}} + w_k \\ &= \sum_{l=1}^{L} \left[e^{-(\alpha_1+j\frac{2\pi}{v_g})\Delta f\Delta d n_l} \right]^k \\ &\quad \cdot \left[g_l \cdot e^{-[(\alpha_0+\alpha_1 f_{\min})+j2\pi\frac{f_{\min}}{v_g}]\Delta d n_l} \right] + w_k. \end{aligned} \tag{22}$$

By defining

$$v_n = e^{-(\alpha_1+\frac{j2\pi}{v_g})\Delta f\Delta d\cdot n}, \quad 1 \le n \le N, \tag{23}$$

$$x_n = \begin{cases} g_l \cdot e^{-[(\alpha_0+\alpha_1 f_{\min})+j2\pi\frac{f_{\min}}{v_g}]\Delta d\cdot n}, & \text{if } n = n_l \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

we could derive a discrete estimation model as,

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{w}', \tag{25}$$

where $\mathbf{y}$, $\mathbf{\Phi}$ and $\mathbf{x}$ are given by,

$$\mathbf{y} = [H_0, H_1, \cdots, H_{M-1}]^T, \tag{26}$$

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ v_1 & v_2 & \cdots & v_N \\ \vdots & \vdots & \ddots & \vdots \\ v_1^{M-1} & v_2^{M-1} & \cdots & v_N^{M-1} \end{bmatrix}_{M\times N}, \tag{27}$$

$$\mathbf{x} = [x_1, x_2, \cdots, x_N]^T, \tag{28}$$

while $\mathbf{w}'$ denotes the noise vector. Here, $\mathbf{x}$ is the target sparse vector containing only L nonzero elements on the corresponding support $\mathcal{T} = \{n_l\}_{l=1}^{L}$, which might finally be solved by the CS theory.

For (25), CS theory has proved that the target signal $\mathbf{x}$ can be exactly recovered by a very small number of observations M if $\mathbf{x}$ is sparse, i.e., the number of nonzero entries of the target signal L is much smaller than its dimension N. In practice, different CS algorithms will have different requirements on the matrix and the sparsity for a reliable recovery. In this paper, we focus on two categories of CS reconstruction: the greedy CS [37], [38] and the Bayesian CS [26]. In the greedy CS algorithms, the restricted isometry property (RIP) of the observation matrix $\mathbf{\Phi}$ is a sufficient condition to guarantee the recovery performance [39]. Unfortunately, there is currently no fast algorithm to check RIP. The common alternative for measuring how well a observation matrix preserves the recovery performance is its coherence $\eta_{\mathbf{\Phi}}$ (the smaller the better) which is defined by the maximum absolute cross-correlation between the normalized columns of $\mathbf{\Phi}$ [39]. One of the well-known results demonstrates that the greedy methods, such as orthogonal matching pursuit (OMP) [37], subspace pursuit (SP) [38] and so on, are guaranteed to perfectly recover the L-sparse vectors when $\eta_{\mathbf{\Phi}} < 1/2L$.

Algorithm 1 RVM Algorithm Based PLC CSI Acquisition

Inputs:
Noisy measurements $\mathbf{y}$; Resolution Δd; Signal dimension N; Target residual r_{th}.

Output: Parameter estimate L, g_l, d_l.

1: ***Initialization:***
Noninformative prior matrix $\mathbf{\Gamma}^0 = \mathbf{I}_N$;
Initial noise variance $\lambda^0 = 0.01 \times \text{var}(\mathbf{y})$ and residual $\mathbf{r}^0 = \mathbf{y}$;
2: Construct the observation matrix $\mathbf{\Phi}$ based on the inputs;
3: $t = 1$;
4: Run the RVM algorithm [25] to update the target signal $\boldsymbol{\mu}_{\mathbf{x}}^t$, the noise variance λ^t, the residual $\mathbf{r}^t$, and the iteration number t until $\|\mathbf{r}^t\|_2 < r_{th}$;
5: $\hat{\mathbf{x}} = S_{3\lambda^t}(\boldsymbol{\mu}_{\mathbf{x}}^t)$; {Blank the element less than $3\lambda^t$.}
6: $\{d_l\}_{l=1}^{L} = \text{supp}(\hat{\mathbf{x}})\Delta d$; {Obtain the electrical length from the nonzero support of target signal.}
7: Calculate g_l according to Eqn. (17).

However, it can be easily seen from (27) that the coherence of the observation matrix $\mathbf{\Phi}$ in our proposed scheme is very large, and hence the performance of the greedy CS algorithms will degrade significantly in this case (see more details in Section VII). The BCS has given us an alternative to solve this problem [26]. The BCS is based on the relevance vector machine (RVM) proposed by Tipping [25], whose idea is to find the posterior probability $f(\mathbf{x}|\mathbf{y};\mathbf{\Omega})$ based on the Bayesian rule. $\mathbf{\Omega}$ denotes the set of the hyperparameters, including the covariance matrix, the noise variance, and etc., which can be derived by marginalizing over $\mathbf{x}$ and performing evidence maximization. Once the hyperparameters are obtained, $\mathbf{x}$ can be estimated via the maximum-a-posterior (MAP) criterion. The BCS algorithms are more robust to the noise since it adopts a statistical model to describe both the sparse signal and the noise, and in the reconstruction process, it incorporates noise statistics in solving the inverse problem. Moreover, unlike the greedy CS solutions, it can handle the matrix with strong coherence, which could well fit our problem [40].

The RVM algorithm based PLC CSI acquisition adopted in this paper is summarized in **Algorithm 1**. After the channel parameters have been obtained, the channel frequency response can be synthesized and used for data equalization.

V. Extension to MIMO PLC

In the MIMO PLC systems, by utilizing the orthogonal pilots, we could have the similar matrix formulation for a certain transmit-receive pair as,

$$\mathbf{y}^{(pq)} = \mathbf{\Phi}\mathbf{x}^{(pq)} + \mathbf{w}^{(pq)}, \quad 1 \leq p \leq N_t,\ 1 \leq q \leq N_r, \tag{29}$$

where

$$\mathbf{y}^{(pq)} = [H_0^{(pq)}, H_1^{(pq)}, \cdots, H_{M-1}^{(pq)}]^T, \tag{30}$$

$$\mathbf{x}^{(pq)} = [x_1^{(pq)}, x_2^{(pq)}, \cdots, x_N^{(pq)}]^T, \tag{31}$$

$$x_n^{(pq)} = g_l^{(pq)} \cdot e^{-[(\alpha_0+\alpha_1 f_{\min})+j(2\pi\frac{f_{\min}}{v_g}+\varphi^{(pq)})]\Delta d \cdot n}, \quad 1 \leq n \leq N. \tag{32}$$

Due to the spatial correlation property of the MIMO PLC channels as mentioned above, the vector $\mathbf{x}^{(pq)}$ to be estimated share the common sparse support, i.e., the indices of the nonzero elements in $\mathbf{x}^{(pq)}$ are the same for all the transmit-receive pairs. By exploiting such inherent property, the multiple measurement vector (MMV) based BCS [27] or block BCS [28] can be applied to enhance the recovery performance. In this section, we will briefly describe both kinds of algorithms and then compare their performances in Section VII.

A. MMV Based BCS

By stacking the noisy measurements $\mathbf{y}^{(pq)}$ into one matrix $\mathbf{Y} = [\mathbf{y}^{(11)}, \mathbf{y}^{(21)}, \cdots, \mathbf{y}^{(N_tN_r)}]$, a matrix formulation of (29) for all antennas can be derived as

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{X} + \mathbf{W}, \tag{33}$$

where $\mathbf{X} = [\mathbf{x}^{(11)}, \mathbf{x}^{(21)}, \cdots, \mathbf{x}^{(N_tN_r)}]$ contains all the target information to be estimated. In this context, the signal recovery problem is extended to a MMV model and can be solved by many BCS methods, one of which, the MSBL algorithm proposed by Wipf and Rao [27], has a satisfying performance with an acceptable complexity. The MSBL algorithm based MIMO PLC CSI acquisition adopted in this paper is summarized in **Algorithm 2**.

However, the existing MMV algorithms typically assume that the target signals are independent and identically distributed processes and ignore the strong correlations of the nonzero element amplitudes. Moreover, if the common path delay assumption become invalid in some cases, as discussed in Section II-B, the MMV BCS based scheme will fail.

B. Block BCS

By incorporating the noisy measurements $\mathbf{y}^{(pq)}$ into an aggregate long vector as

$$\dot{\mathbf{y}} = \left[(\mathbf{y}^{(11)})^T, (\mathbf{y}^{(21)})^T, \cdots, (\mathbf{y}^{(N_tN_r)})^T\right]^T, \tag{34}$$

the matrix formulation of (33) can be rewritten as

$$\dot{\mathbf{y}} = \begin{bmatrix} \mathbf{\Phi} & & & \\ & \mathbf{\Phi} & & \\ & & \ddots & \\ & & & \mathbf{\Phi} \end{bmatrix}_{N_tN_rM \times N_tN_rN} \begin{bmatrix} \mathbf{x}^{(11)} \\ \mathbf{x}^{(21)} \\ \vdots \\ \mathbf{x}^{(N_tN_r)} \end{bmatrix}_{N_tN_rN \times 1} \triangleq \mathbf{D}\dot{\mathbf{x}} + \dot{\mathbf{w}}. \tag{35}$$

Algorithm 2 MSBL Algorithm Based MIMO PLC CSI Acquisition

Inputs:
Noisy measurements $\mathbf{Y}$; Resolution Δd; Signal dimension N; Target residual r_{th}.
Output: The parameter estimate L, $g_l^{(pq)}$, d_l.
1: ***Initialization:***
Noninformative prior matrix $\mathbf{\Gamma}^0 = \mathbf{I}_N$;
Initial noise variance $\lambda^0 = 0.01 \times \text{var}(\mathbf{Y})$ and residual $\mathbf{R}^0 = \mathbf{Y}$;
2: Construct the observation matrix $\mathbf{\Phi}$ based on the inputs;
3: $t = 1$;
4: Run the MSBL algorithm [27] to update the target signal $\mathcal{M}_{\mathbf{X}}^t$, the noise variance λ^t, the residual $\mathbf{R}^t$, and the iteration number t until $\|\mathbf{R}^t\|_{\mathcal{F}} < r_{th}$;
5: $\hat{\mathbf{X}} = S_{3\lambda^t}(\mathcal{M}_{\mathbf{X}}^t)$; {Blank the element less than $3\lambda^t$.}
6: $\{d_l\}_{l=1}^L = \text{supp}(\hat{\mathbf{x}}^{(11)})\Delta d$; {Obtain the electrical length from the nonzero support of target signal.}
7: Calculate $g_l^{(pq)}$ according to Eqn.(32).

Then, by rearranging the elements of the aggregate target vector $\dot{\mathbf{x}}$ as $\mathbf{z} = [\mathbf{z}_1^T, \mathbf{z}_2^T, \cdots, \mathbf{z}_N^T]^T$ with $\mathbf{z}_n = [x_n^{(11)}, x_n^{(21)}, \cdots, x_n^{(N_tN_r)}]^T$, the system model of (35) can be rewritten as

$$\dot{\mathbf{y}} = \mathbf{\Psi}\mathbf{z} + \dot{\mathbf{w}}, \tag{36}$$

where

$$\mathbf{\Psi} = \left[\boldsymbol{\phi}_1 \otimes \mathbf{I}_{N_tN_r}, \boldsymbol{\phi}_2 \otimes \mathbf{I}_{N_tN_r}, \cdots, \boldsymbol{\phi}_N \otimes \mathbf{I}_{N_tN_r}\right]_{N_tN_rM \times N_tN_rN}, \tag{37}$$

and $\boldsymbol{\phi}_i$ is the i-th column vector of matrix $\mathbf{\Phi}$.

Thanks to the spatial correlation, the rearranged aggregate target vector $\mathbf{z}$ exhibits block sparsity, leading to an additional structural constraint on the system model. Similarly, due to the strong coherence of the composition matrices $\mathbf{\Phi}$ in $\mathbf{\Psi}$, $\mathbf{\Psi}$ demonstrates a bad RIP as well which cannot guarantee the recovery performance of the conventional greedy CS algorithms for block sparse signals. The block BCS could solve this problem and achieve enhanced performance by introducing a block covariance matrix $\mathbf{B}$ to describe the inherent correlation within the blocks. In this paper, we propose a modified BSBL algorithm for MIMO PLC CSI acquisition, which can work despite the common path delay assumption, as summarized in **Algorithm 3**.

This scheme is based on the standard BSBL algorithm, but has the following differences.

1) *Adaptivity:* Unlike to the standard algorithm where halting criterion is setting the residual to a fixed threshold, we also add a constraint of $\|\mathbf{r}^t\|_2 < \|\mathbf{r}^{t-1}\|_2$. In this way, the algorithm could be more adaptive and robust in various channel conditions, for example, if there is strong interference caused by the partial different path delays.

2) *Blanking Operation:* In the proposed scheme, the output of the standard BSBL algorithm $\boldsymbol{\mu}_{\mathbf{z}}^t$ will first be blanked and then the nonzero support will be obtained for the final

Algorithm 3 Modified BSBL Algorithm Based MIMO PLC CSI Acquisition

Inputs:
Noisy measurements $\dot{\mathbf{y}}$; Resolution Δd; Signal dimension N; Target residual r_{th}.
Output: The parameter estimate L, $g_l^{(pq)}$, $d_l^{(pq)}$.
Initialization:
Noninformative prior matrix $\mathbf{\Gamma}^0 = \mathbf{I}_N$;
Initial block covariance matrix $\mathbf{B} = \mathbf{I}_{N_t N_r}$, noise variance $\lambda^0 = 0.01 \times \mathrm{var}(\mathbf{z})$, and residual $\mathbf{r}^0 = \dot{\mathbf{y}}$;
Construct the observation matrix $\mathbf{\Psi}$ based on the inputs.
$t = 1$;
while $\|\mathbf{r}^t\|_2 < \|\mathbf{r}^{t-1}\|_2$ or $\|\mathbf{r}^t\|_2 > r_{th}$ **do**
  $\mathbf{\Sigma}_0 = \mathbf{\Gamma}^{t-1} \otimes \mathbf{B}$;
  $\mathbf{\Xi}_{\dot{\mathbf{y}}}^t = (\mathbf{\Sigma}_0{}^{-1} + \frac{1}{\lambda^{t-1}}\mathbf{\Psi}^H\mathbf{\Psi})^{-1}$; {Update the covariance matrix of the target signal.}
  $\boldsymbol{\mu}_{\mathbf{z}}^t = \mathbf{\Sigma}_0\mathbf{\Psi}^H(\lambda^{t-1}\mathbf{I} + \mathbf{\Psi}\mathbf{\Sigma}_0\mathbf{\Psi}^H)^{-1}\dot{\mathbf{y}}$; {Update the target signal.}
  $\gamma_i^t = \frac{1}{N_t N_r}\mathrm{Tr}[\mathbf{B}^{-1}(\mathbf{\Xi}_{\dot{\mathbf{y}},i}^t + \boldsymbol{\mu}_{\mathbf{z},i}^t(\boldsymbol{\mu}_{\mathbf{z},i}^t)^H)]$ for $i = 1, 2, \ldots, N$; {Learning rule for the block scalars.} ($\boldsymbol{\mu}_{\mathbf{z},i}^t$ is the i-th block in $\boldsymbol{\mu}_{\mathbf{z}}^t$ and $\mathbf{\Xi}_{\dot{\mathbf{y}},i}^t$ is the i-th principal diagonal block in $\mathbf{\Xi}_{\dot{\mathbf{y}}}^t$.)
  $\mathbf{\Gamma}^t = diag(\gamma_1^t, \gamma_2^t, \cdots, \gamma_N^t)$;
  $\mathbf{r}^t = \dot{\mathbf{y}} - \mathbf{\Psi}\boldsymbol{\mu}_{\mathbf{z}}^t$; {Update the residual.}
  $\lambda^t = (\|\mathbf{r}^t\|_2^2 + \mathrm{Tr}(\mathbf{\Xi}_{\dot{\mathbf{y}}}^t\mathbf{\Psi}^H\mathbf{\Psi}))/N$; {Learning rule for the noise variance.}
  $\mathbf{B} = \frac{1}{N}\sum_{i=1}^{N}(\mathbf{\Xi}_{\dot{\mathbf{y}},i}^t + \boldsymbol{\mu}_{\mathbf{z},i}^t(\boldsymbol{\mu}_{\mathbf{z},i}^t)^H)/\gamma_i^t$; {Learning rule for the block covariance matrix $\mathbf{B}$.}
  $t \leftarrow t + 1$;
end while
$\mathcal{D} = \mathrm{supp}\left(S_{3\lambda^t}(\boldsymbol{\mu}_{\mathbf{z}}^t)\right)$; {Blank the element less than $3\lambda^t$ and obtain the support.}
$\hat{\mathbf{z}}|_{\mathcal{D}} = (\mathbf{\Psi}|_{\mathcal{D}})^\dagger\,\dot{\mathbf{y}}$; {Final LS estimation of the target signal.}
$\{d_l^{(pq)}\}_{l=1}^L$ can be obtained from the support of $\hat{\mathbf{z}}$;
Calculate $g_l^{(pq)}$ according to Eqn.(32).

LS estimation. This operation could further reduce the possible error of estimation due to the partial different delays in MIMO PLC channels.

3) *Initialization:* In certain systems, the parameters, such as the covariance matrix, the informative prior matrix, the threshold, the noise distribution and so on, could be specified initially and thus reduce the computational complexity.

VI. Theoretical Analysis and Discussions

A. Complexity Analysis

The computational complexity of the sparse CSI acquisition approaches for PLC systems is compared in this subsection. According to [24], the annihilating filter equation is a Yule-Walker system and does not include the specific noise suppression operation. In the noise-free case, it needs $2L$ measurements and a complexity of $\mathcal{O}(L^2)$ for accurate estimation. In the noisy case, the method uses oversamplings of M instead of $2L$ ($M > 2L$) to overcome the noise and is solvable in $\mathcal{O}(M^2)$ with M noisy measurements.

The computational complexity of the ESPRIT based scheme is mainly based on the SVD operation which is in the order of $\mathcal{O}(L^3)$ in the noise-free case. In the noisy case, the measurement number increases to M, and the complexity is $\mathcal{O}(M^2L + L^3)$ via some simplified implementation.

The complexity of the proposed RVM, MSBL, and BSBL based schemes is mainly based on the matrix inversion operation. For RVM based scheme, the complexity of matrix inversion operation in (25) usually should be $\mathcal{O}(N^3)$, but by implementing an iterative formula, the cost can be reduced to $\mathcal{O}(M^2N)$. Similarly, the complexity of the MSBL and BSBL based schemes are $\mathcal{O}(M^2N)$ and $\mathcal{O}(N_t^3N_r^3M^2N)$, respectively.

B. Discussion on Assumptions and Limitations

Before adopting the proposed scheme for sparse CSI acquisition, we would like to discuss the assumptions in the derivations above.

1) The CSI acquisition problem is focusing on the estimation of two parameters, i.e., g_l and d_l. The other parameters including the group velocity v_p and the attenuation factors α_0, α_1 are assumed to be constant, which indicates the homogeneity of the power cable. However, the power line network may consist of different cables and the homogeneity will be no longer valid. In practice, we can use the averaged parameters instead.

2) By defining a finite resolution Δd, the continuous estimation problem in (2) and (6) is turned into a discrete estimation problem at the cost of accuracy degradation. Fortunately, in the simulations, we will show such method has superior noise robustness and the possible error floor cause by discretization is acceptable in practical systems.[4]

VII. Simulation Results

In this section, we will provide the simulations to validate the the proposed scheme. Besides, some remarks and discussions are also addressed.

A. Simulation Setup

For the SISO case or the PN-PN channel in MIMO cases, the L-path random PLC channel model is adopted for evaluation [41]. For the other channels in MIMO cases, the model in (5) is adopted for simulation according to [11], [33], and [34], where the values of $\varphi^{(pq)}$ are drawn randomly according to a uniform distribution between $-\Delta\varphi/2$ and $\Delta\varphi/2$, and $\Delta\varphi = \pi$ represents the average correlation with the PN-PN channel. The channel paramters are listed in Table I.

The other simulation parameters are summarized in Table II. It should be noted that due to the proposed system model in (2) and (5), the pilots adopted in this paper is the regularly combed pilots which locate uniformly in the sub-carriers.

B. Recovery Probability

To evaluate the performance of the proposed scheme and the greedy CS (OMP) based scheme, Fig. 4 presents the

[4]The practical PLC systems have adopted some efficient channel coding scheme for reliable transmission. In this case, the error floor in the MSE of CSI acquisition will no longer be a serious issue as long as its accuracy at the certain SNR could reach the required MSE for successful decoding.

TABLE I
PARAMETERS OF THE RANDOM PLC CHANNEL

Parameter	Notation	Value or Model
Path Number	L	10
Path Electrical Length	$d_l, l = 1, \cdots, L$	Poisson Arrival Process with intensity $\Lambda = 0.2m^{-1}$ and maximum signal length $d_{\max} = 800m$
Path Gain	$g_l^{(pq)},\ l = 1, \cdots, L$ $p = 1, 2,\ q = 1, 2, 3$	Uniform Distribution on $[-1, 1]$
Correlation Parameter	$\varphi^{(pq)}, p = 1, 2, q = 1, 2, 3$	Uniform Distribution on $[-\pi/2, \pi/2]$
Group Velocity	v_g	$2 \times 10^8 m/s$
Attenuation Parameters	α_0, α_1	$\alpha_0 = 0, \alpha_1 = 7.8 \times 10^{-10} s/m$
Electrical Length Resolution	Δd	$1m$

TABLE II
PARAMETERS FOR SIMULATIONS

Parameter	Notation	Value
Signal Frequency Band	$[f_{\min}, f_{\max}]$	$[2, 30]$ MHz
Guard Interval Length	N_G	256
OFDM Data Block Length	N_D	2048
Pilot Number Per Transmit Antenna	M	64

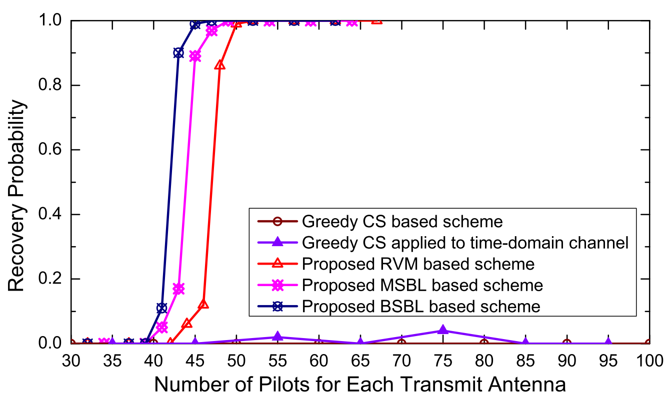

Fig. 4. The recovery probability of the proposed schemes and the greedy CS based scheme at the SNR = 20 dB.

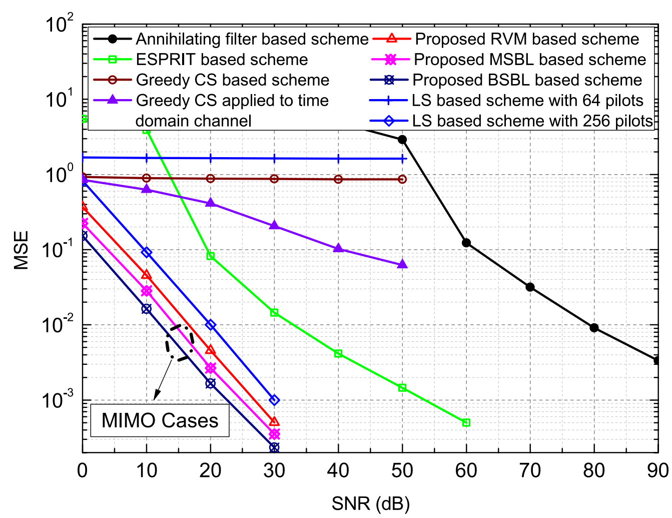

Fig. 5. The MSE performance comparison of the proposed schemes and the related schemes.

correct channel recovery probability when different numbers of pilots M are used with the fixed SNR of 20 dB. The scheme that directly applies OMP into the time-domain PLC channel with the iteration number equal to $3L$ is also performed for comparison, whereby the randomly distributed pilots are adopted to guarantee the RIP of the observation matrix [42]. The correct recovery is defined as the estimation mean square error (MSE) is lower than 10^{-2}, and here the MSE is given by

$$\mathrm{MSE} = \sqrt{\frac{\int_{f_{\min}}^{f_{\max}} |H(f) - \bar{H}(f)|^2 \mathrm{d}f}{\int_{f_{\min}}^{f_{\max}} |H(f)|^2 \mathrm{d}f}}, \tag{38}$$

where $\bar{H}(f)$ is the synthetic channel frequency response with the estimated parameters.

It can be noticed from Fig. 4 that by utilizing the spatial correlation, the required number of the pilots M for reliable recovery could be reduced from around $M = 52$ to $M = 48$ compared to the SISO case. Besides, the BSBL algorithm could well exploit the inherent correlation structure of the MIMO PLC channels, and hence could further reduce the required pilot number per transmit antenna. The theory also gives the number of observations M in the BCS algorithm should be no less than $L \log(N/L) = 44$ for a reliable recovery [25]. Besides, the greedy CS cannot work well in either our proposed model or the time-domain PLC channel model.

In practice, considering the tradeoff between the robustness of the channel recovery performance and the pilot overhead, we choose the pilot number for each transmit antenna $M = 64$ for the proposed scheme in our following simulations.

C. Estimation Performance Comparison

Fig. 5 illustrates the MSE performance of different schemes. The performances of the greedy CS (i.e., OMP [37]) based scheme is presented, while the scheme that directly applies

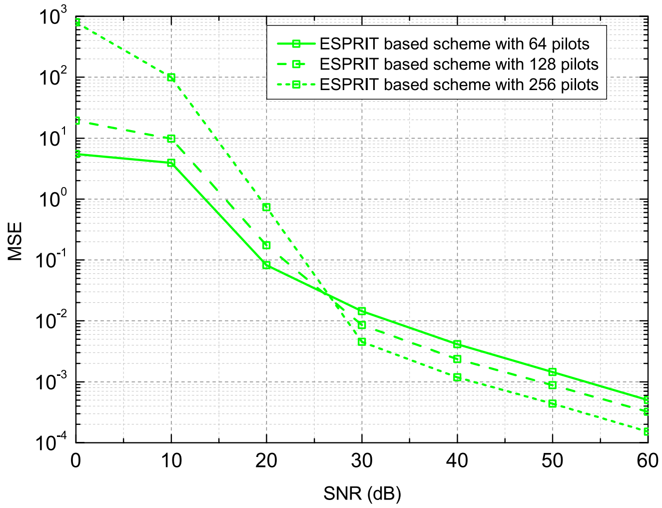

Fig. 6. The MSE performance comparison of the ESPRIT scheme with different pilot numbers.

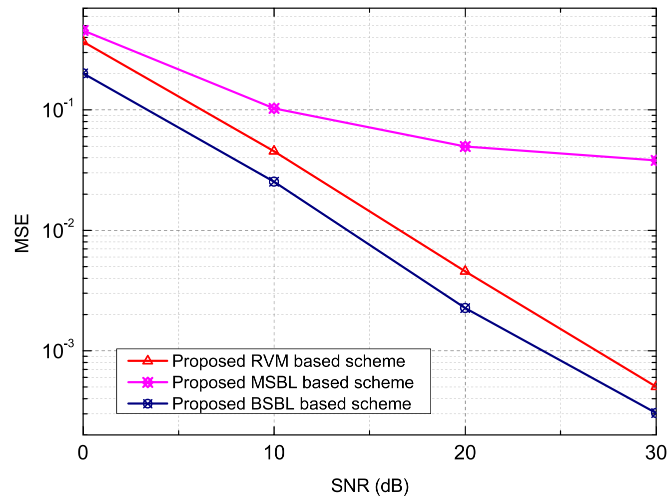

Fig. 7. The MSE performance comparison if there only exist partial common path delays.

OMP into the time-domain channel is also performed for comparison. The conventional LS based scheme usually needs much more measurements, and here both 256 and 64 comb-type pilots are adopted in this scheme for simulation. It could be seen that the proposed RVM based scheme demonstrates better MSE performance than the conventional counterparts. The annihilating filter based and ESPRIT based schemes need very high SNR to reach a considerable MSE (10^{-2}), which are infeasible for practical systems. The greedy CS based scheme has failed completely due to the strong coherence of the observation matrix, of which the estimation performance is even worse than that of the straightforward application of greedy CS to time-domain channel. In MIMO cases, by utilizing the channel spatial correlation, the proposed MSBL based scheme could achieve a performance improvement of around 2.5 dB than the RVM based SISO scheme, while the proposed modified BSBL based scheme could further enhance the performance of 2.5 dB compared to the MSBL based one.

We have also carried out the simulations on the ESPRIT based scheme with different pilot numbers, as illustrated in Fig. 6. It can be seen from Fig. 6 that the performance of the ESPRIT based scheme will improve with the increasing pilot number only when the SNR is high, i.e., above a certain threshold. Here, the threshold is around 27 dB under our simulation configurations, which is too high and impractical in communication systems. The performance of the ESPRIT based scheme with low SNR is still unreliable.

For the MIMO channels that might not share the same path delay support (we assume here they have 8 common path delays), simulations are carried out to evaluate the proposed schemes as well, as depicted in Fig. 7. In such scenarios, the MSBL based scheme will degrade significantly while the modified BSBL based scheme can still work well.

Moreover, the spectral efficiency of the proposed scheme and the conventional LS based scheme with standard 256 pilots for the SISO case has been specified in Table III according to the criterion specified in [38], and the proposed scheme has much higher spectral efficiency.

TABLE III
THE SPECTRAL EFFICIENCY COMPARISON

Solution	Spectral Efficiency
Conventional LS based scheme	77.8%
Proposed Sparse CSI Acquisition Scheme	86.1%

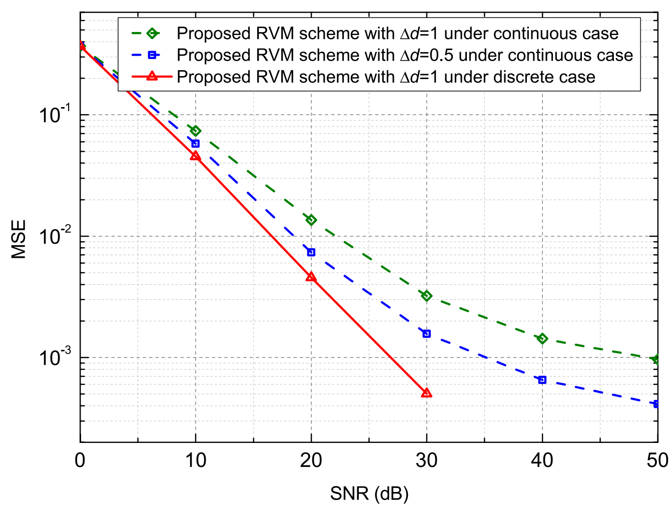

Fig. 8. The MSE performance comparison of the proposed scheme in discrete and continuous cases.

D. Discussion on the Discretization

It can be seen from the derivation part of the above schemes that the annihilating filter method and ESPRIT algorithm cope with the continuous signal estimation problem, while the CS theory works on a discrete model. Consequently, by discretizing the electrical length and applying the BCS in our proposed scheme, the estimation performance will suffer from potential degradation. To verify the performance degradation caused by discretization, we modify the simulated electrical length from integers to random real numbers in order to model the real continuous signals. For simplicity, the SISO case is used as an example. The MSE performance of different cases with different electrical length resolutions is presented in Fig. 8. We find that for the continuous case (more close to the real channel condition), the proposed RVM based scheme will degrade and have an error floor due to the finite resolution. In practice, such error floor will not become a serious issue due to the deployment of powerful channel coding. It means that a reliable transmission could be maintained as long as the error floor of channel estimation is lower than the required MSE for successful decoding by choosing a proper resolution. From Fig. 8, the MSE of the scheme with $\Delta d = 1m$ can achieve 3×10^{-3} at SNR of 30dB, which is accurate enough to support the reliable transmission modulated with 256QAM constellation [43] and indicates that $\Delta d = 1m$ can be a good choice as the resolution under such transmission requirements.

VIII. Conclusions

In this paper, by exploiting the parametric sparsity and discretizing the electrical length in the well-known PLC channel model, we formulate the non-sparse (either time-domain or frequency-domain) model into a sparse problem. Furthermore, we propose a noise-robust and spectrally efficient CSI acquisition scheme under the framework of BCS, and extend it to the MIMO PLC channels. The proposed scheme has better MSE performance and spectral efficiency than the conventional schemes, including both the annihilating filter based, ESPRIT based schemes and the classical LS based scheme. We believe the proposed scheme might be a promising solution for practical PLC systems.

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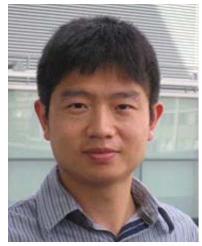

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