LETTER

Narrowband Interference Mitigation Based on Compressive Sensing for OFDM Systems

Sicong LIU[†a)], *Nonmember*, Fang YANG[†b)], *Member*, Chao ZHANG[†c)], *Nonmember*, *and* Jian SONG[†,††d)], *Member*

SUMMARY A narrowband interference (NBI) estimation and mitigation method based on compressive sensing (CS) for communication systems with repeated training sequences is investigated in this letter. The proposed CS-based differential measuring method is performed through the differential operation on the inter-block-interference-free regions of the received adjacent training sequences. The sparse NBI signal can be accurately recovered from a time-domain measurement vector of small size under the CS framework, without requiring channel information or dedicated resources. Theoretical analysis and simulation results show that the proposed method is robust to NBI under multi-path fading channels.

key words: *narrowband interference (NBI), compressive sensing (CS), repeated training sequences (TS)*

1. Introduction

Narrowband interference (NBI) exists in many communication systems, such as the NBI generated by Bluetooth in the IEEE 802.11 wireless local area network (WLAN) system, and the NBI caused by crosstalk or radio frequency in digital subscriber line or power line communication (PLC) systems. Among the conventional NBI mitigation schemes, a frequency threshold excision (FTE) approach by detecting and nulling out the sub-carriers whose powers exceed a certain threshold is proposed in [1]. Hard decisions of the orthogonal frequency division multiplexing (OFDM) symbols are adopted to predict the NBI contribution over the used sub-carriers to sequentially subtract the NBI impairment [2], while the estimation error of one sub-carrier would be propagated to all subsequent sub-carriers.

For sparse signal reconstruction, a variable step size normalized least mean square method is proposed in [3]. Recently, the a ground-breaking theory of compressive sensing (CS) proves that a sparse signal can be recovered effectively through a measurement vector of much smaller size than the signal dimension in the presence of noise. CS has been widely investigated in various areas [4]. A null-space (NS) approach firstly introduced CS theory to NBI mitigation. With the aid of channel estimation, the NS of the channel transfer matrix is calculated to obtain the measurement vector for the recovery of the sparse NBI signal [5].

In the CS framework, it is important to acquire the measurement vector for the CS-based recovery of the sparse NBI signal. However, the data or TS components whose powers are much higher than AWGN are not tolerable in the measurement vector and should be nulled out before performing the CS algorithm, which is a crucial issue in the CS-based NBI mitigation. Hence, in this letter, we propose a low-complexity CS-based differential measuring (CSDM) method for NBI mitigation in communication systems. We simply obtain the measurement vector by the differential operation between the inter-block-interference (IBI)-free regions of the adjacent received training sequences (TSs) with low complexity, and then reconstruct the NBI based on the CS theory accurately. We also propose a threshold-based support adjustment method for the estimated support of the CS algorithm to further improve the accuracy. The proposed approach enjoys significant performance gain over conventional methods for the OFDM systems impacted by NBI.

Notation: Throughout this letter, boldface uppercase and lowercase letters are used to denote matrices and column vectors, respectively; $(\cdot)^\dagger$ denotes the Moore-Penrose matrix pseudo-inversion; $\|\cdot\|_q$ denotes the ℓ_q norm operator; $\mathbf{x}|_\Omega$ denotes the entries of the vector $\mathbf{x}$ in the set Ω. Finally, Ψ_Ω represents the sub-matrix comprised of the Ω columns of the matrix Ψ.

2. System Model

As shown in Fig. 1, many communication systems have adopted repeated TSs for channel estimation, synchronization, etc. For example, as shown in Fig. 1(a), two identical TSs are adopted as the preamble in IEEE 802.11n WLAN systems. As shown in Fig. 1(b), the identical TSs in term of the so-called UW are used in UW-SC systems. Also in time-domain synchronous OFDM (TDS-OFDM) systems as shown in Fig. 1(b), repeated TSs are used as guard intervals between OFDM data blocks [6], [7].

In communication systems with repeated TSs, we take TDS-OFDM as shown in Fig. 1(b) as a typical example without loss of generality. The ith symbol consists of the constant TS $\mathbf{c} = [c_0, c_1, \cdots, c_{M-1}]^T$ of length M and the following OFDM data block $\mathbf{x}_i$ of length N, where the TSs for

Manuscript received June 25, 2014.
Manuscript revised November 28, 2014.

[†]The authors are with the Research Institute of Information Technology, Department of Electronic Engineering, Tsinghua National Laboratory of Information Science and Technology (TNList), Tsinghua University, Beijing 100084, P.R. China.
[††]The author is also with National Engineering Lab. for DTV (Beijing), Beijing 100191, P.R. China.
a) E-mail: liu-sc12@mails.tsinghua.edu.cn
b) E-mail: fangyang@tsinghua.edu.cn
c) E-mail: z_c@tsinghua.edu.cn
d) E-mail: jsong@tsinghua.edu.cn

DOI: 10.1587/transfun.E98.A.870

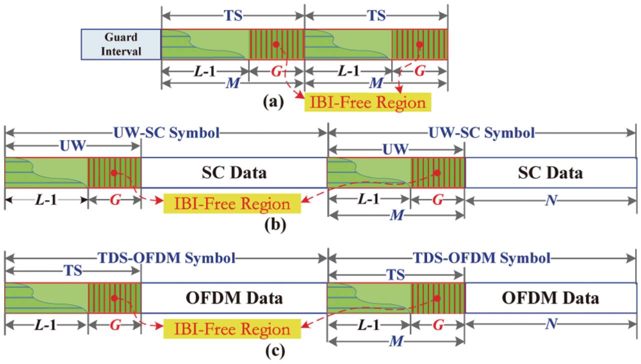

Fig. 1 Typical examples of repeated TSs in communication systems: (a) WLAN 802.11 Preamble; (b) UW-SC symbol; (c) TDS-OFDM symbol.

different symbols are identical. Then the transmitted signal passes through the multipath channel with the channel impulse response (CIR) of $\mathbf{h} = [h_0, h_1, \cdots, h_{L-1}]^T$ of length L in the presence of NBI. Since the $(i-1)$th OFDM data block $\mathbf{x}_{i-1}$ only causes IBI on the first $L-1$ samples of the ith TS, the last $G = M - L + 1$ samples of the ith TS form the IBI-free region. The IBI-free region exists in practical systems because a common rule for system design is to configure the guard interval length M to be larger than the maximum channel delay spread L in the worst case to avoid IBI [8]. Hence the two time-domain IBI-free regions at the end of the two adjacent received TSs can denoted by

$$\mathbf{y}_i = \mathbf{\Phi}_G\mathbf{h}_i + \mathbf{F}_G\tilde{\mathbf{e}}_i + \mathbf{w}_i, \tag{1}$$

$$\mathbf{y}_{i+1} = \mathbf{\Phi}_G\mathbf{h}_{i+1} + \mathbf{F}_G\tilde{\mathbf{e}}_{i+1} + \mathbf{w}_{i+1}, \tag{2}$$

where $\mathbf{y}_i$ and $\mathbf{w}_i$ denote the received IBI-free region vector and the additive white Gaussian noise (AWGN) vector whose each entry has zero mean and variance σ^2, respectively, while the inverse Fourier transform matrix $\mathbf{F}_G$ of size $G \times N$ is defined as $\mathbf{F}_G = \frac{1}{\sqrt{N}}[\boldsymbol{\varphi}_0 \quad \boldsymbol{\varphi}_1 \quad \cdots \quad \boldsymbol{\varphi}_{N-1}]$, where the mth entry of the column $\boldsymbol{\varphi}_k$ is $\exp(j2\pi km/N)$ for $m = M-G, M-G+1, \cdots, M-1$. The convolution between the CIR and the TS is denoted by $\mathbf{\Phi}_G\mathbf{h}_i$, where the partial Toeplitz matrix $\mathbf{\Phi}_G$ is given by

$$\mathbf{\Phi}_G = \begin{bmatrix} c_{L-1} & c_{L-2} & c_{L-3} & \cdots & c_0 \\ c_L & c_{L-1} & c_{L-2} & \cdots & c_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{M-1} & c_{M-2} & c_{M-3} & \cdots & c_{M-L} \end{bmatrix}_{G\times L}. \tag{3}$$

The frequency-domain NBI vector, which is a sparse signal [1], is denoted by $\tilde{\mathbf{e}}_i = [e_{i,0}, e_{i,1}, \cdots, e_{i,N-1}]^T$ with only a few nonzero entries. The support Ω of the NBI, whose sparsity level $S = |\Omega| \ll N$, is the positions set of the nonzero entries that are randomly distributed within all of the N possible sub-carriers. The average power of the NBI is $P_e = \sum_{k\in\Omega} |e_{i,k}|^2/S$. Hence, the interference-to-noise ratio (INR) used to measure the strength of the NBI can be defined as $P_e\big/\sigma^2$.

3. CS-Based NBI Recovery and Mitigation

It is crucial to null out the data and TS components to acquire the measurement vector for accurate CS-based NBI recovery. The proposed CSDM approach is performed by the following steps:

3.1 Step I. Differential Measuring for NBI

Based on the fact that the duration of each transmission frame is sufficiently small, the CIR for the adjacent two TSs is assumed to be quasi-static, i.e., $\mathbf{h}_i \approx \mathbf{h}_{i+1}$. Therefore, we can null the TS components by subtracting (2) from (1), i.e., through the differential measuring on the IBI-free regions, and the CS measurement equation can be simply obtained as

$$\Delta\mathbf{y}_i = \mathbf{F}_G\Delta\tilde{\mathbf{e}}_i + \Delta\mathbf{w}_i, \tag{4}$$

where

$$\Delta\mathbf{y}_i = \mathbf{y}_i - \mathbf{y}_{i+1}, \tag{5}$$

$$\Delta\tilde{\mathbf{e}}_i = \tilde{\mathbf{e}}_i - \tilde{\mathbf{e}}_{i+1} = [\Delta e_{i,0}, \Delta e_{i,1}, \cdots, \Delta e_{i,N-1}]^T, \tag{6}$$

$$\Delta\mathbf{w}_i = \mathbf{w}_i - \mathbf{w}_{i+1}. \tag{7}$$

$\Delta\tilde{\mathbf{e}}_i$ is the NBI differential vector. Also because the frame duration is sufficiently small, the NBI signal between two adjacent TSs can be regarded as invariant and the time-domain NBI vector of the $(i+1)$th TS equals to that of the ith TS delayed by Δd samples, where $\Delta d = M + N$ is the distance between the two adjacent received TSs. Hence the frequency-domain NBI vector $\tilde{\mathbf{e}}_{i+1}$ relates to $\tilde{\mathbf{e}}_i$ with a constant phase shift $2\pi k\Delta d/N$, and then we have

$$\Delta e_{i,k} = e_{i,k}(1 - e^{\frac{j2\pi k\Delta d}{N}}), \quad k = 0, 1, \cdots, N-1. \tag{8}$$

Now it is feasible to recover the unknown NBI differential vector $\Delta\tilde{\mathbf{e}}_i$ from the measurement vector $\Delta\mathbf{y}_i$ by solving the underdetermined problem (4) based on the CS theory.

Since only the simple differential operation is adopted, the computational complexity $O(M)$ of the proposed CSDM method is quite low. Besides, it can be verified through numerical calculation analysis that the observation matrix $\mathbf{F}_G$ of the proposed CSDM approach satisfies the restricted isometry property (RIP) well with the $2S$-RIP constant $\delta_{2S} < 0.39$ for $S = 4$, which proves that accurate NBI recovery can be achieved.

3.2 Step II. NBI Recovery Using SAMP

After CSDM, the formulated problem (4) can be mathematically converted to the following convex optimization problem

$$\min_{\Delta\tilde{\mathbf{e}}_i\in\mathbb{C}^N} \|\Delta\tilde{\mathbf{e}}_i\|_1, \quad \text{s.t.} \quad \left\|\Delta\mathbf{y}_i' - \mathbf{F}_G\Delta\tilde{\mathbf{e}}_i\right\|_2 \le \varepsilon, \tag{9}$$

where ε is the noise bound for the AWGN vector $\Delta\mathbf{w}_i'$. It has been proved that (9) can be efficiently solved as long as $G > O(S\log_2(N/S))$ using classical CS algorithms, such as orthogonal matching pursuit (OMP) [9] and sparsity adaptive matching pursuit (SAMP) [10], to recover the NBI differential vector $\Delta\tilde{\mathbf{e}}_i$. We adopt SAMP which is more appropriate for the realistic NBI model since it does not require the sparsity level to be known.

The input of SAMP includes the measurement vector $\Delta\mathbf{y}_i$, the observation matrix $\mathbf{\Psi} = \mathbf{F_G}$, and the iteration step size Δs. The output is the recovered NBI differential vector $\Delta\hat{\mathbf{e}}_i$ satisfying $\Delta\hat{\mathbf{e}}_i|_{\Omega_0} = \mathbf{\Psi}^{\dagger}_{\Omega_0}\Delta\mathbf{y}_i$, and $\Delta\hat{\mathbf{e}}_i|_{\Omega_0^c} = \mathbf{0}$, where Ω_0 and Ω_0^c denote the estimated support of $\Delta\hat{\mathbf{e}}_i$ and its complementary set respectively. The main idea of SAMP is to achieve the final estimated NBI vector through stage-wise greedy iterations. In each iteration, the previous estimated support is merged with indices maximizing the correlations between the columns of the observation matrix and the residual, and the estimated support is updated by the projection of the measurement vector on the merged support. The step size Δs could be adjusted according to NBI strength and occurrence probability. Finally, the iterations halt when the norm of the final residual $\mathbf{r}$ is less than the noise bound described in (9), i.e., $\|\mathbf{r}\|_2 = \|\Delta\mathbf{y}_i - \mathbf{\Psi}\Delta\hat{\mathbf{e}}_i\|_2 \approx \|\Delta\mathbf{w}_i\|_2 < \varepsilon$, where ε can be configured according to the noise distribution [10].

3.3 Step III. Accuracy Improvement and Final Cancellation

Since the sparsity level is unknown in practice, the estimated support Ω_0 of SAMP might be not accurate enough, especially when the INR is relatively low or the channel condition is poor. Hence, we propose a threshold-based support adjustment method to further improve the estimation accuracy of the NBI support. The refined support Ω_{th} includes the entries whose ℓ_2 norms are larger than the threshold $\gamma_{\text{th}} = \alpha\log\left(\hat{P}_e/\sigma^2\right)\hat{P}_e$, where the estimated average NBI power is $\hat{P}_e = (1/N)\sum_{k=0}^{N-1}\left|\Delta\hat{e}_{i,k}\right|^2$, $\Delta\hat{e}_{i,k}$ is the kth entry of $\Delta\hat{\mathbf{e}}_i$, and α is a coefficient which can be set proportional to the NBI strength. Then the recovered NBI differential vector $\Delta\hat{\mathbf{e}}_i$ of SAMP can be updated at the refined support Ω_{th} such that $\Delta\hat{\mathbf{e}}_i|_{\Omega_{th}} = \mathbf{\Phi}^{\dagger}_{\Omega_{th}}\Delta\mathbf{y}_i$ and $\Delta\hat{\mathbf{e}}_i|_{\Omega_{th}^c} = \mathbf{0}$. Furthermore, the NBI values at the refined support can be more accurate through least squares (LS) estimation by solving the following

$$\min_{\Delta\hat{\mathbf{e}}_i\in\mathbb{C}^N} \|\Delta\mathbf{y}_i - \mathbf{\Pi}\Delta\hat{\mathbf{e}}_i\|_2, \tag{10}$$

where $\mathbf{\Pi} = \mathbf{F}_G\mathbf{B}$, and $\mathbf{B}$ is the $N \times N$ diagonal selection matrix whose elements $b_{k,k} = 1$ for $k \in \Omega_{\text{th}}$ and zero otherwise. Finally, the recovered NBI differential vector is given by

$$\Delta\hat{\mathbf{e}}_i = \mathbf{B}\mathbf{\Pi}^{\dagger}\Delta\mathbf{y}_i. \tag{11}$$

With the recovered NBI differential vector $\Delta\hat{\mathbf{e}}_i$, we can now reconstruct the original frequency-domain NBI vector $\tilde{\mathbf{e}}_i$ as

$$e_{i,k} = \Delta\hat{e}_{i,k}/(1 - e^{j\frac{2\pi}{N}k\Delta d}), \quad k = 0, 1, \cdots, N-1. \tag{12}$$

Afterwards, by just adding a phase shift, the frequency-domain NBI vector $\tilde{\mathbf{e}}_i^D = \left[e^D_{i,0}, e^D_{i,1}, \cdots, e^D_{i,N-1}\right]^T$ corresponding to the ith OFDM data block can be given by

$$e^D_{i,k} = e_{i,k}\exp(j2\pi kM/N), \quad k = 0, 1, \cdots, N-1. \tag{13}$$

After cancelling the finally recovered frequency-domain NBI vector $\tilde{\mathbf{e}}_i^D$ from the received OFDM data block, the transmitted data immune from NBI can be obtained and then decoded.

4. Simulation Results

Simulations are carried out to evaluate the performance of the proposed NBI mitigation scheme. The TDS-OFDM system is adopted as a representative of communication systems with repeated TSs. We set the OFDM data block length $N = 512$, the TS length $M = 127$, and $\alpha = 2.5$. The modulation constellation is 64QAM, and a low density parity check (LDPC) code with a block length of 8,640 bits and a code rate of 0.5 is applied as the forward error correction code [11]. The Vehicular-A multi-path channel model [8] is adopted for evaluation.

The mean square error (MSE) performance comparison of the proposed CSDM scheme is shown in Fig. 2. The Cramér-Rao lower bound (CRLB) is also presented for performance comparison, where CRLB $= 2\sigma^2(SN/G)$ [8]. The proposed CSDM method achieves an MSE less than 10^{-2} when the INR of the NBI signal reaches 25.8 dB and 34.7 dB at the sparsity level $S = 2$ and $S = 4$, respectively. It is noted that the MSE performance approaches the CRLB with the increase of INR.

Figure 3 shows the LDPC-coded bit error rate (BER) performance under Vehicular-A channel in the presence of NBI with INR of 30 dB and sparsity level of $S = 4$. The BER performance of the conventional FTE method, the NS method, the ideal case without NBI, and the case ignoring NBI are also depicted for comparison. It can be observed

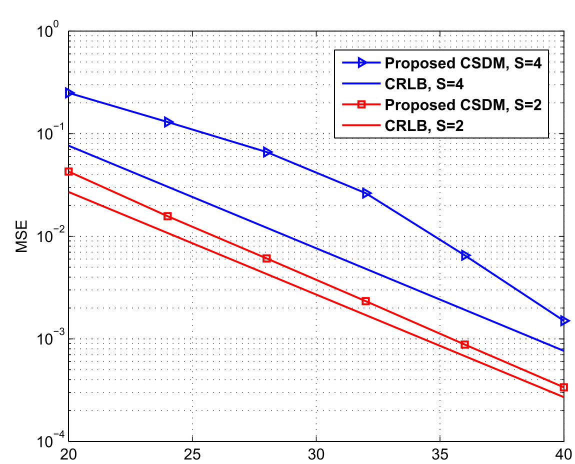

Fig. 2 MSE performance comparison under the Vehicular-A channel.

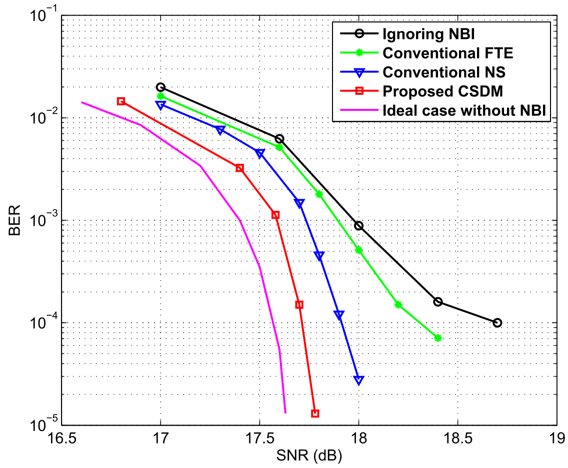

Fig. 3 BER performance comparison under the Vehicular-A channel.

that the proposed CSDM method enjoys about 0.62 dB and 0.23 dB gain over conventional FTE and NS methods at the target BER of 10^{-4}, respectively. We can also observe that the CSDM method significantly outperforms the case ignoring NBI by about 1.0 dB, and performs only about 0.14 dB away from the ideal case without NBI.

5. Conclusion

In this letter, a novel CS-based NBI mitigation method is proposed for communication systems with repeated TSs. The sparse NBI signal can be accurately recovered through the CS-based differential measuring between the IBI-free regions of the adjacent received TSs with very low complexity, without requiring channel information or other dedicated resources. Simulation results demonstrate that the proposed method significantly outperforms the conventional methods, and performs close to the ideal case without NBI. The proposed scheme could find many applications in practice based on the fact that repeated TSs are widely adopted by various communication systems, such as the preamble in the IEEE 802.11n standard, and the ITU G.hn PLC standard, etc.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant No. 61401248, 61471219).

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